

Example 1

Why do people go to the sauna when they want to lose weight?

The heat of vaporization of the water is 2,3 MJ per kg. So by vaporization of one liter of water we will lose 2,3 MJ.

But believing that you can lose weight in sauna is absolutely incorrect. A sauna does not help you to lose weight; it temporarily removes easily replaceable water from the body. Saunas have been traditionally used to produce a feeling of relaxation. As your heart rate goes up and your blood vessels dilate, there is an increase in blood flow to the skin. Saunas may also improve blood circulation. The sweating process gently and safely helps eliminate toxins in the body. Medical studies demonstrate that most toxins can be eliminated through the skin, relieving the burden on the kidneys and liver.

Example 2

We often hear that we lose a lot of heat from our head. Let us do some math to see if that is true. We call it testing the hypothesis.

The human body can be roughly divided into several components:

Head: Considered as a hemisphere.

Trunk: Approximated as a cylinder.

Arms: Approximated as two cylinders (upper and lower arms).

Legs: Approximated as two cylinders (thighs and lower legs).

Hands and feet: We will estimate them as rectangles

Let's calculate the surface area of each part:

Head:

Most caps have circumference 55 cm(C), so we will use for the head surface calculation $A = 4\pi r^2$, $C = 2\pi r$, hence $A = \frac{C^2}{\pi} = 0,096m^2$, so let us use **0,1m²**.

Trunk:

The average circumference of the trunk can be around 0.75 meters (based on an average waist size), so the radius $r = \frac{C}{2\pi} = 0,12m$

The height of the trunk can be approximately 0.5 meters.

Surface area of a cylinder $= 2\pi r h + 2\pi r^2 = Ch + 2\pi r^2 = 0,375m^2 + 0,092m^2 =$
0,465 m²

Arms:

Average length of the upper arm = ~0.25 meters.

Average length of the lower arm = ~0.25 meters.

Average circumference of upper arm = ~0.2 meters, radius = 0,03m.

Average circumference of lower arm = ~0.18 meters, so let us do an approximation for 0,2 meters.

$$\text{Surface area of each arm} = 2\pi rh + 2\pi r^2 = 0,05m^2 + 0,006m^2 = 0,056m^2$$

$$\text{Surface area of both arms} = \mathbf{0,11m^2}$$

Legs:

Average length of the thigh = ~0.4 meters.

Average length of the lower leg = ~0.4 meters.

Average circumference of thigh = ~0.35 meters, radius 0,06m.

Average circumference of lower leg = ~0.25 meters, radius 0,04m.

$$\text{Surface area of lower leg} = 2\pi rh + 2\pi r^2 = 0,1m^2 + 0,01m^2 = 0,11m^2$$

$$\text{Surface area of the thigh} = 2\pi rh + 2\pi r^2 = 0,14m^2 + 0,02m^2 = 0,16m^2$$

$$\text{Surface area of both legs} = \mathbf{0,55 m^2}$$

Hands and feet:

Surface area of one hand is approximately $15 \text{ cm}^2 = 0,0015 \text{ m}^2$, two hands **0,003 m²**

Surface area of one foot is approximately $375 \text{ cm}^2 = 0,0375 \text{ m}^2$, two feet **0,075 m²**

Now, sum up the surface areas of all these parts:

$$\text{Total surface area} \approx (0,465 + 0,11 + 0,55 + 0,003 + 0,075)m^2 = 1,2 m^2$$

So, based on this approximation, the surface area of an average adult human body is approximately 1,3 square meters. Remember that this is just an estimate and individual variations can occur. Now if we put it in percents:

Head accounts for about 7,5% of the body's surface area.

Trunk: 35,7%

Arms: 8,5 %

Legs: 42,3 %

Hands and feet: 0,2% + 5,8%

Now, to our calculation. Imagine that you are wearing clothing that covers your entire body, except for your head.

The outside temperature is freezing, around 0°C.

Your body's core temperature is a normal 37°C.

Body lose heat by:

Convection – around 15 %

Radiation - around 60%

Evaporation – around 25%

When the body is in direct contact with an object that is cooler or hotter than we must also take conduction into account.

But for now, we will focus on radiation, which is the biggest contributor.

Imagine losing heat through radiation like the warmth you feel from a campfire. This happens all over the body, not just the head.

The formula for figuring out how much heat we lose through radiation is a bit complicated but it involves these things: how hot we are, how cold it is around us, and how much of our skin is exposed,

The formula is called Stefan-Boltzmann Law:

$$Q = \sigma A(T_1^4 - T_2^4)$$

Where:

Q is the amount of heat we lose (in watts, W).

σ is Stephan – Boltzmann constant, its value is $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

A is how much skin is exposed (in square meters, m^2).

T_1 is the temperature of our body (in kelvin, K).

T_2 is the temperature around us (in kelvin, K).

Heat loss through the head by radiation:

$$Q = \sigma A(T_1^4 - T_2^4) = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} 0,1 \text{ m}^2 (310,15^4 \text{ K}^4 - 273,15^4 \text{ K}^4) \cong 20 \text{ W}$$

Heat loss through the rest of the body:

$$Q = \sigma A(T_1^4 - T_2^4) = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} 1,2 \text{ m}^2 (310,15^4 \text{ K}^4 - 273,15^4 \text{ K}^4) \cong 250 \text{ W}$$

In winter, we usually wear warm clothes. Even if our body is not covered, heat loss through the head is only about 7%.

Let's assume the clothing provides good insulation and reduces heat loss by 90%.

In that case, we would only lose about 25 W.

So, in this example, you still lose more heat through the covered body than through the uncovered head. It's important to wear appropriate clothing to minimize overall heat loss in cold weather and yes cover your head as well.

Summary:

Hypothesis: We initially proposed that the majority of body heat is lost through the head.

Testing the Hypothesis:

We began by calculating the surface area of the head and other body parts. To simplify these calculations, we used estimations based on recognizable shapes, making math more manageable.

We conducted a comprehensive study of the various ways the body loses heat.

Our primary focus was on the calculations for heat loss through radiation, as it is the most significant contributor to overall heat loss.

Our calculations revealed that heat loss through radiation, which we expected to be the largest portion, was actually only 20W from the head, while the rest of the body accounted for 250W of heat loss.

Conclusion: Our original hypothesis, which suggested that the head is the primary source of heat loss, was disproven by our calculations. In the scientific community, results are shared and published for other scientists to scrutinize and test. Only after a consensus is reached among experts can a hypothesis become widely accepted as an objective truth. This process illustrates the rigorous and collaborative nature of scientific inquiry.

Example 3

How good are cars in extracting energy from fuel?

We will consider a car that moves in a straight line with constant speed $90\text{km}\cdot\text{h}^{-1}$. According to the law of inertia, it should remain in this movement even without having to burn some fuel. So why a car actually needs some fuel. Let us break it down.

Forces that slow the car down

Cars have to deal with two types of things that slow them down:

External forces

These are like invisible things that resist the car's movement. Two big ones are air resistance and rolling friction.

Internal Losses

These happen inside the car when different parts rub against each other, making it less efficient.

To keep the car moving, it needs a steady supply of energy to overcome these forces.

What we are looking at

We'll focus on the outside forces and ignore the stuff happening inside the car.

Air resistance force

The basic data for our car will be from the Technical Certificate. Golf GTI 2.0 has a maximum power 221 kW (300k), cylinder displacement 1984 cm³, fuel gasoline, mass

1498 kg standby + 532kg permissible load including operator, highest permitted speed 250 km / h, consumption 8,4 l/100 km in the city, 5,6 l/100 km out of city. Acceleration from 0 to 100 km / h 5,6 s, CO₂ emissions 151g / km, air resistance coefficient $c_x = 0.31$).

Think about how the wind pushes against you when you stick your hand out of a moving car.

The formula for calculation was determined by experiments and observations by Newton

$$F_{air\ resistance} = \frac{1}{2} \rho c_x A_x v^2$$

ρ is the density of air – $1,28 \frac{kg}{m^3}$

air resistance coefficient $c_x = 0.31$ is from Technical Certificate

A_x is front area of the car – our approximation is $2 m^2$. (Now you can understand why sports car have so small frontal area.)

$$F_{air\ resistance} = \frac{1}{2} 1,28 \frac{kg}{m^3} 0,31 \cdot 2m^2 (25m \cdot s^{-2})^2 = 248 N$$

Rolling friction force

When your cars tires roll on the road, there is some resistance. We calculate this as:

$$F_{rolling\ friction} = mgc_r = 2030 kg \cdot 10m \cdot s^{-2} \cdot 0,0098 = 200N$$

The rolling resistance factor c_r depends on the degree of wheel inflation, the tread thickness, the hardness of the tire, the type of surface on which the car is driven, and many other factors. For standard situation, $c_r = 0,0098$.

Work done by both forces in 100 km

$$W = (248 + 200)N \cdot 100\ 000m = 44,8 \cdot 10^6 J$$

Energy gained from fuel for 100 km

In the tables we can find that the energy released from 1 l of fuel is 61 MJ = $61 \cdot 10^6 J$

$$E = 61 \cdot 10^6 J \cdot 1^{-1} \cdot 5,6 l = 341,6 \cdot 10^6 J$$

How efficient is the car?

To find out how well the car turns fuel into power, we can calculate its efficiency:

Efficiency = (Useful Energy Output) / (Total Energy Input) x 100%

$$\text{efficiency} = \frac{44,8 \cdot 10^6 J}{341,6 \cdot 10^6 J} 100\% = 13\%$$

In our case, the car's efficiency is about 13%.