Mechanics of Fluids – Additional Material

Tasks to find out:

1.Galileo's thermometer and its connection to Archimedes' Principle

2. Archimedes' claw, screw, mirrors used to win the battle against Romans

Numerical examples.

Example 1.

Weighing the car with a pressure gauge.

With this example you will need help of a driver in your family. You will need car, pressure gauge, ruler, big sheets of paper.

For weighing the car, we will use the formula: $p = \frac{F}{A} = \frac{m_{car} g}{A}$

$$m_{car} = \frac{p_1\,(pressure\ in\ the\ 1.tyre).\,A_1(area\ of\ the\ 1.tyre) + p_2\,A_2 + p_3\,A_3 + p_4\,A_4}{g}$$

Now you need to measure the pressure in each tire and the area with which the tyre touches the road. To calculate the area place 4 pieces of paper on the ground around the tire. Then measure the lengths of the sides of the rectangle with a ruler and calculate the area. Repeat the procedure for each tire.

Now you can do the calculation and compare it with the car data.

Example 2.

When ice melts, the level of water is a) higher b) the same c) lower

$F_B = \rho_{water}\, V_{submerged\ part\ of\ ice}\, g = mg$ → mg is weight of ice

$F_B$ - equals the weight of water from the hole, the object "made" in water.

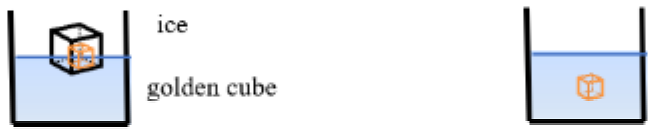
$\rho_{water} = 1000\ kg/m^3$

$\rho_{ice} = 916\ kg/m^3$

b.) is correct

An ice cube floats in water because its weight is compensated by the buoyant force. According to Archimedes: buoyant force = weight of displaced water. Melting does not change the weight of ice, which means: weight of ice = weight of melted water. According to all written above, we get: weight of melted water = weight of displaced water. From there it follows: volume of melted water = volume of displaced water. Well, if we also realize that the volume of displaced water is nothing but the volume of the submerged part of the ice, we finally get: volume of melted water = volume of the submerged part of the ice.

Having golden cube in the ice, after melting of the ice the level of water is a) higher b) the same c) lower.




$\rho_{water}$ = 1000 kg/m$^3$

$\rho_{gold}$ = 19320 kg/m$^3$

$F_B = \rho_{water} V_{\text{submerged part}}\, g = (m_{ice} + m_{gold})\, g$

What happens to the golden cube → it sinks (its density is higher than the density of water)

We already know that ice filled its own hole in the water.

Let's have:

1 kg of ice,

100 g (0,1 kg) of gold,

it will displace 1,1 kg of water.

1 kg of ice displaces 1kg of water.

100 g of gold in water displace only the amount of water with the same volume as gold.

$$V_{gold} = \frac{0{,}1\ kg}{19320\ \frac{kg}{m^3}} = 0{,}000\ 0052 \frac{kg}{\frac{kg}{m^3}} = 0{,}000\ 0052 \frac{kg.m^3}{kg} = 0{,}000\ 0052\ m^3$$

this volume will displace:

$m_{water} = 1000\frac{kg}{m^3}\ .0{,}000\ 0052 m^3 = 0{,}0052\ kg$

$m_{\text{water}}$ = 0,0052 kg, which is less than 0,1 kg.

c) is correct

When in ice it displaced the amount of water equal to its mass.

When in water it displaced the amount of water equal to its volume.

Example 3.

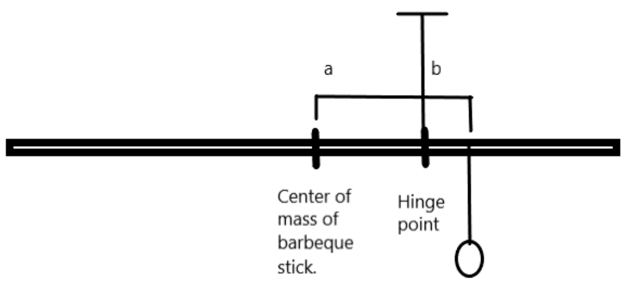
Measuring the density of golden ring using barbeque sticks.

Find the center of the barbeque stick

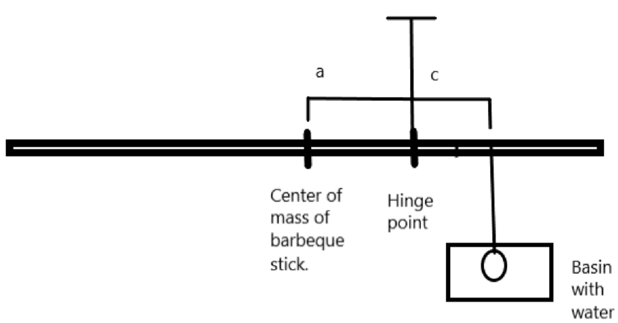
Hang the stick 5 cm from the center to one side -*a*.

Balance the stick by hanging a gold ring on fine nylon.

Measure the distance *b* of the nylon from the hinge point.




Immerse the ring into the water - the balance is disturbed.




So again, we move the nylon until it is in balance - distance *c*.

In air: $F_{g(\text{stick})} \cdot a = F_{g(\text{ring})} \cdot b$

In water: $F_{g(\text{stick})} \cdot a = (F_{g(\text{ring})} - F_{B(\text{ring})}) \cdot c$

where $F_{B(\text{ring})}$ is the buoyant force. From above written equations we get:

$$F_{g(\text{ring})} \cdot b = (F_{g(\text{ring})} - F_{B(\text{ring})}) \cdot c$$

$$m_{(ring)} \cdot b = \left(m_{(ring)} - \frac{m_{(ring)}}{\rho_{ring}} \rho_{water}\right) \cdot c$$

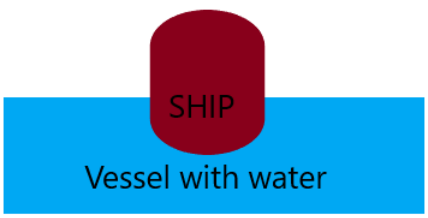
from this we just express $\rho_{ring}$, $m_{(ring)}$ falls out:

$$\rho_{ring} = \frac{\rho_{water}}{\left(1 - \frac{b}{c}\right)}$$

<u>Example 4.</u>

Is there a minimum required height for a ship to float?

Let us calculate a simplified example.




What is the height of the ship 's draft?

Weight of the ship: measure

Volume of the ship: $V = Ah$ ($h$=height)

Area ( in our simplified example): $A = \pi r^2$, measure the diameter.

Height of the ship 's draft $h_i$: $mg = V_{immersed\ part}\, \rho g$

$$h_i = \frac{m}{\rho \pi r^2}$$

What is the minimum height of water level? Do we really need the water level up to the height of the ship 's draft ? If we already have water in the container before placing the ship inside, the water level will increase. Remember: the ship displaces an amount of water equal to the volume of its submerged portion. Therefore, the necessary quantity of water will be the volume of water in the container up to a height of "$h_i$" minus the volume of the ship up to the same height of "$h_i$.

Interesting way how to help imagine the situation was presented in:

https://vnuf.cz/sbornik/prispevky/17-30-Onderova.html

You'll need two see-through plastic cups, one little bit smaller, so that we can slide them into each other (e.g. 300 ml and 350ml). 300ml should be empty, and the other 350 ml has water in it, but not filled to the very top. Mark the level of water.

Place the empty cup upside down on the water in the other cup. It floats on top of the water.

Add a little water to the empty cup and put it back on top of the water. It still floats, but this time, it's slightly below the water's surface.

Pour almost all the water from the full cup into the second cup, leaving just a tiny bit of water in the bottom of the first cup. The second cup still floats, and no water spills out.

Repeat this experiment a few times, and you'll notice that the water levels in both cups pretty much stay the same.

So, why does this happen? It's all thanks to buoyancy. Remember Archimedes' law says that an object floats in water if the buoyancy force exerted on it by the fluid balances its weight. Buoyant force is equal to the weight of the fluid displaced by submerged part of the body.

The cups are super light compared to the water, so it doesn't make much difference.  This means they don't push away much water. So, the water levels in both cups stay the same.

This experiment teaches us that even a small amount of water can make things float, as long as there's enough space for the water to move out of the way. If you tried this with a much larger container, the cup might not float because it wouldn't be able to push enough water out of the way.

Example 5.

Explain how the density of a body fully immersed in water can be determined by weighing the body in air and in a liquid (water).

$\rho$ - density of the liquid

$\rho_1$- unknown body density

$F_g$ - body weight in air

$F$ - body weight in the liquid (weighing of the body immersed in the liquid)

$F_B$ – buoyant force

$$F = F_g - F_B$$

$$F_B = F_g - F$$

$$\rho g V = F_g - F\ ;\ \rho_1 = \frac{m}{V} \rightarrow V = \frac{m}{\rho_1}$$

$$\rho g V = F_g - F$$

$$mg\frac{\rho}{\rho_1} = F_g - F$$

$$F_g\frac{\rho}{\rho_1} = F_g - F$$

$$F_g\rho = \rho_1\left(F_g - F\right)$$

$$\rho_1 = \frac{F_g\rho}{\left(F_g - F\right)}$$

<u>Example 6.</u>

The standard states that in a submarine for one sailor there must be an average space $V_1 = 20$ m$^3$. How many sailors can work in a submarine if a buoyant force $F_B$ = 15,3 MN acts on the submarine when immersed in seawater ($\rho$ = 1020 kg.m$^{-3}$).

Given: $\rho = 1020\,\frac{\mathrm{kg}}{\mathrm{m}^3}$; $V_1 = 20\ \mathrm{m}^3$; $F_B = 15{,}3\ \mathrm{MN} = 15\ 300\ 000\ \mathrm{N}$

$$F_B = \rho V g$$

$$V = \frac{F_B}{\rho g}$$

$$V = \frac{15\ 300\ 000\ \mathrm{N}}{1020\,\frac{\mathrm{kg}}{\mathrm{m}^3}\,10\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 1500\ \mathrm{m}^3$$

$$n = \frac{V}{V_1} = \frac{1500\ \mathrm{m}^3}{20\ \mathrm{m}^3} = 75$$

According to the standard there can be 75 sailors.

<u>Example 7.</u>

The density of sea water $\rho = 1030$ kg.m$^{-3}$, the density of ice $\rho_i = 915$ kg.m$^{-3}$. What percentage of the glacier protrudes above the open sea level? The ones who have seen the film Titanic know what problem it may cause.

Given: $\rho = 1030\,\frac{\mathrm{kg}}{\mathrm{m}^3}$ ; $\rho_i = 915\,\frac{\mathrm{kg}}{\mathrm{m}^3}$

$$m_i g = \rho g V_{immersed}$$

$$\rho_i V = \rho V_{immersed}$$

$$V_{immersed} = \frac{\rho_i V}{\rho}$$

$$V_{immersed} = \frac{915\,\frac{\mathrm{kg}}{\mathrm{m}^3}}{1030\,\frac{\mathrm{kg}}{\mathrm{m}^3}} V = 0{,}89\,V$$

$$x = V - V_{immersed} = V - 0{,}89V = 0{,}11\,V = 11\%\,V$$

$11\%$ of the glacier protrudes above the open sea level.

Example 8.

The Bernoulli effect can have significant implications for building design. One notable consequence is the potential for high-speed winds to create low pressure around a skyscraper. This contrast in pressure can lead to windows popping out due to the higher atmospheric pressure within the building. An actual incident occurred at the John Hancock Building in Boston, where wind caused windowpanes to be dislodged and fall several stories to the sidewalk below. Suppose there is a horizontal wind blowing at a speed of 11,2 m/s outside a large plate glass pane measuring 4,00 m × 1,50 m. We can assume that the air density remains constant at 1,20 kg/m$^3$, and that the air inside the building is at atmospheric pressure. What is the total force exerted by the air on the windowpane?

Given: $v = 11{,}2\,\frac{\mathrm{m}}{\mathrm{s}}$ ; window dimension $4{,}00\ \mathrm{m} \times 1{,}50\ \mathrm{m}$; $\rho = 1{,}20\,\frac{\mathrm{kg}}{\mathrm{m}^3}$

$$p_{outside} + \frac{1}{2}\rho {v_{outside}}^2 = p_{inside} + \frac{1}{2}\rho {v_{inside}}^2$$
$$p_{outside} + \frac{1}{2}\rho {v_{outside}}^2 = p_{inside} + \frac{1}{2}\rho \left(0\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2$$
$$\frac{1}{2}\rho {v_{outside}}^2 = p_{inside} - p_{outside}$$

$$\frac{1}{2}\rho {v_{outside}}^2 A = (p_{inside} - p_{outside})A$$
$$\frac{1}{2}\rho {v_{outside}}^2 A = F$$
$$F = \frac{1}{2}\,1{,}20\,\frac{\mathrm{kg}}{\mathrm{m}^3}\left(11{,}2\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2 (4{,}00\ \mathrm{m} \times 1{,}50\ \mathrm{m})$$
$$F = 451{,}6\ \mathrm{N}$$

Force exerted on the window pane was 451,6 N.

Example 9.

In the wider part of the tube, water flows at a speed of $v_1$ = 10 cm.s$^{-1}$. At what speed does the water flow in its narrower part, which has 2 times smaller radius.
Given: $v_1 = 10\,\frac{\mathrm{cm}}{\mathrm{s}}, r_2 = \frac{r_1}{2}$

$$A_1 v_1 = A_2 v_2$$
$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi {r_1}^2}{\pi {r_2}^2} v_1 = \frac{{r_1}^2}{\left(\frac{r_1}{2}\right)^2} v_1 = 4 v_1 = 4.10\,\frac{\mathrm{cm}}{\mathrm{s}} = 40\,\frac{\mathrm{cm}}{\mathrm{s}} = 0{,}4\,\frac{\mathrm{m}}{\mathrm{s}}$$

In the narrower part the water flows with the speed $40\,\frac{\mathrm{cm}}{\mathrm{s}} = 0{,}4\,\frac{\mathrm{m}}{\mathrm{s}}$

<u>Example 10.</u>

This example is the solution of the example presented by the video in the chapter Mechanics of Fluids, when solving the task of the snorkel's length

To remind you the facts.

We need 2-3 m long transparent (plastic) hose. We will make the form of a U-tube and fill it partially with water. We blow the air in one arm by mouth, overpressure in the mouth can be determined from the difference of heights *h* levels in both vertical tubes. *p* = *ρgh*, where *ρ* is the water density, *g* is gravity acceleration. Will there be a difference if we suck the air?

According to the video the hight of the column when breathe out was 870 mm and for breathe in it was 820 mm. Will we be able to snorkel comfortably in such depth?

Let us do the calculation now:

We will do the calculation for the height 870 mm, the area of the chest 30cmx20cm and the density of fresh water $\rho = 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}$.

$$p = \frac{F}{A} \Rightarrow F = pA = \rho g h A = 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 0{,}87\ \mathrm{m} \cdot 0{,}3\ \mathrm{m} \cdot 0{,}2\ \mathrm{m} = 522\ \mathrm{N}$$

This force of 522 newtons means that it feels like there's about 52 kilograms of weight on your chest, back, and both sides. So, it would be easier to breathe out because you would feel squeezed from all sides.

What we did now, was the calculation of the pressure of the water level above the diver. But there is also the air pressure above the water level. We have already mentioned that we live surrounded by air, but we don't feel it. However, changes in pressure can affect us when we dive into water or go up a mountain.

These pressure differences can also cause problems for fish and divers. When deep-sea organisms are brought to the surface, they face serious issues. If they have any gas space in their bodies, like a swim bladder, the gas will expand due to the pressure difference between the deep sea and the surface, which can harm or kill the animal.

One specific problem divers face is called "caisson disease." When diving, the weight of the water causes increased pressure on the diver's body. To counteract this, divers use special equipment and a breathing gas mixture. However, nitrogen from the air they breathe dissolves in their body tissues due to the high pressure. The deeper and longer they stay underwater, the more nitrogen their bodies absorb. When they ascend to the surface, the pressure decreases, and the excess nitrogen can form bubbles in the body tissues and blood vessels.

These bubbles can lead to a condition called decompression sickness or caisson disease. It can happen if the diver rises too quickly, stays underwater for too long, flies in an aircraft shortly after diving, or it can happen to miners leaving a mine where the pressure was high. Even astronauts can be affected when they leave their space modules and work in space because the pressure inside their spacesuits is lower than the pressure in the module.

Example 11

How to make a cup of justice.

In our book, we introduced the concept of the „Cup of Justice.“ We provided instructions for creating it using a  3D printer. Now, we want to offer an easy way for everyone to make at home.

**Materials needed:**

- 2 plastic cups
- straw with the joint
- glue gun,
- soldering iron for making holes in the bottle and lid,
- instead of soldering iron you can also use heated metal reusable straw,
- gloves.

**Procedure:**

1. Using the soldering iron, make a hole in the bottom of both cups. It's important that these holes connect when you turn the cups toward each other.

2. Bend the straw at the joint and thread it through the hole you created. Ensure that the joint remains inside the cup.

3. Secure the straw in place and simultaneously seal it with the hot glue gun.

4. Secure the straw in place and simultaneously seal it with the hot glue gun. Pass the protruding part of the straw through the hole in the second cup.

**Conclusion**

When we pour water into the Pythagoras cup over the end of the straw, the water will flow out through the straw due to the cohesion of the water and the effect of hydrostatic pressure,. The siphon in the bathroom works on the same principle.

Source: https://vida.cz/blog/pythagoruv-pohar