MECHANICS OF FLUIDS

ARPHYMEDES

Blaise Pascal (1623 – 1662)

Fig. 31

With his brother in – law demonstrated that air pressure changed depending on the altitude.
Blaise Pascal (1623- 1662) lived in the 17th century. His mother died when he was three years old, so he was raised by his father, who was a highly educated man. They lived in Paris, where he met famous scientists. He wrote his first scientific work on sound at the age of eleven. When he was 18, he designed the first calculating machine. As a tribute to his talent the programming language was named after him PASCAL. Pascal was the first to clarify the cause of why mercury at Torricelli's experiment rises to a maximum of 76 cm. He also examined the compressibility of liquids and the distribution of pressure in them. He also conducted public experiments. 31-year-old Pascal was involved in a horse carriage accident, and this influenced his intellectual interests, causing him to become more religious.

Archimedes (287 -212 BCE)

Discovered how to prove, whether the king's crown was from pure gold or not measuring the upthrust of displaced water.
Archimedes was born in Syracuse on the island of Sicily. The city was a Greek colony back then. Little is known about the life of Archimedes but luckily, we know much more about his inventions. Archimedes was the first one who combined the two basic elements of science, experimental research, and mathematics. There is a famous story mentioned in all his biographies. Once a messenger came to Archimedes giving him the task from the King. Archimedes had to test the honesty of the goldsmith. Whether the crown he made was really of pure gold. Archimedes was thinking about the task in the bath as he often did. As he was forgetful, he filled the bath up to the brim. He entered it and some amount of water overflowed. Watching it Archimedes already knew how to test the honesty of the goldsmith. He ran out naked, shouting Eureka.

Fig. 32

Daniel Bernoulli (1700 – 1782)

Fig. 33

The pressure of fluid drops when it is moving.
He belonged to the second generation of the Bernoulli family of Swiss mathematicians. He investigated not only mathematics but also such fields as medicine, biology, physiology, mechanics, physics, astronomy, and oceanography. He is particularly remembered for his applications of mathematics to mechanics, especially fluid mechanics, and for his pioneering work in probability and statistics. Bernoulli's principle, a particular example of the conservation of energy is named after him. His personal life was marked by rivalry and his father's jealousy of his son's achievements.

On this page you can find some proposals for projects that you can make at school or at home. There are even more on the accompanying webpage.

Project proposals:

1. Save the egg. Find a way how an egg can fall from the 1st floor without any harm. You cannot wrap it to prevent breaking
2. Make a cardboard ship, using cardboard, aluminum foil for the bottom, scissors and sellotape. The aim is to find a shape, so that the ship can bear 2kg load and must float in a sink.
3. Make a boat that will be powered by something you can find at home. Use plastic bottles, cans to make the ship.
4. Using the Bernoulli principle, make a hoovercraft with Flexible Corrugated Pipe Sleeve
5. See how cup of justice work. If you have 3D printer, you can make it for yourself. Devious Pythagorean Cup by jsteuben - Thingiverse

Technical applications

Do you know:

1. Why airplanes fly?
2. How submarines work?
3. How water-lock works?

Find information, explanations, and figures on the Internet, books or ask parents, teachers to help you. Then you can discuss it in the classroom.

When preparing some project, it is good to decide on a plan. The important points are:

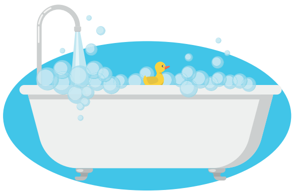
1. Conduct some research, investigate various books, search the Internet, consult your teacher or parent. Write down any comments, or questions.
2. Prepare your proposal, outlining the materials you will need and the procedure you intend to follow. Use as many pictures as possible.
3. Consider how you can test the results.
4. Record, analyze and test your product.
5. Reflect on potential improvements for your product. If possible, implement any necessary changes.

Fluid mechanics is the study of fluids

Low pressure

High pressure

Blaise Pascal
1623-1662



Paddle of water

Archimedes
287-212 BC

Hydrostatics

Mechanics of fluids

We pour water from bottle to glass, or from one glass to another glass. This is what we do daily. But can we do the same with gas?

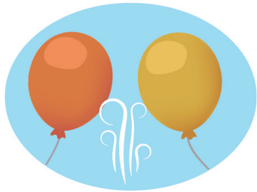
See how we can pour gas. Think about the necessary property of gas that enabled us to do this experiment.

Your comments, question, observations.

- At rest, especially when under pressure. This is called **hydrostatics**.
- In motion. This is called **hydrodynamics**.

Hydrodynamics

Daniel Bernoulli
1700-1782



Blow in
the middle

Fluids can be either liquids or gasses. Let us deal with some properties of fluids. We will explore them doing the following experiments.

We all know air. We need it for our life. But there are many other gasses. For example, carbon dioxide we breathe out. What will happen when we pour carbon dioxide on the lighted candle?

Why did we use carbon dioxide to test whether gasses can flow?

Arphy's questions:

- Was there a difference between pressing water and air in the syringe?
- Why was it not possible to press the syringe with water?

I will give you a hint. The answers have to do with compressibility of fluids.

Your comments, question, observations.

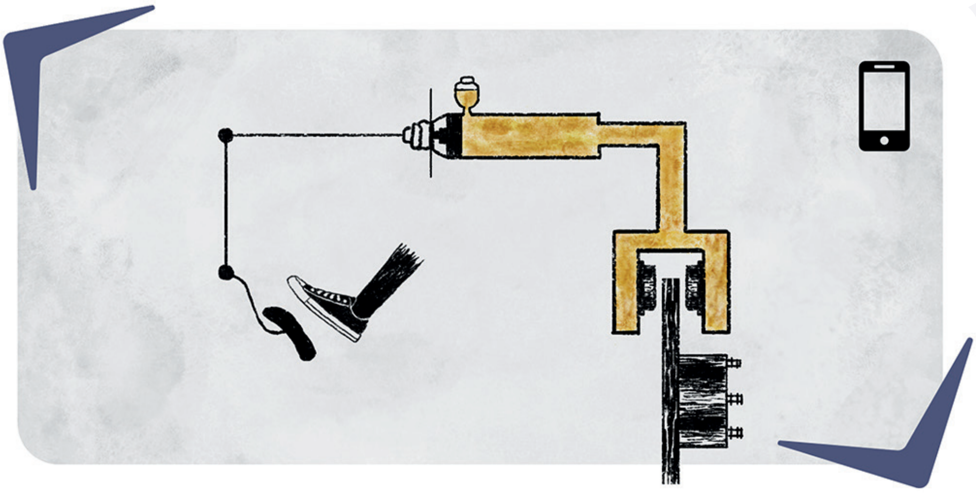
So far, we tested that both liquids and gasses (fluids) can flow. In what situations act liquids and gasses differently? Let us apply pressure on them. You will only need a syringe.

Take a syringe and see what happens when pressing the piston of syringe filled with air and water.

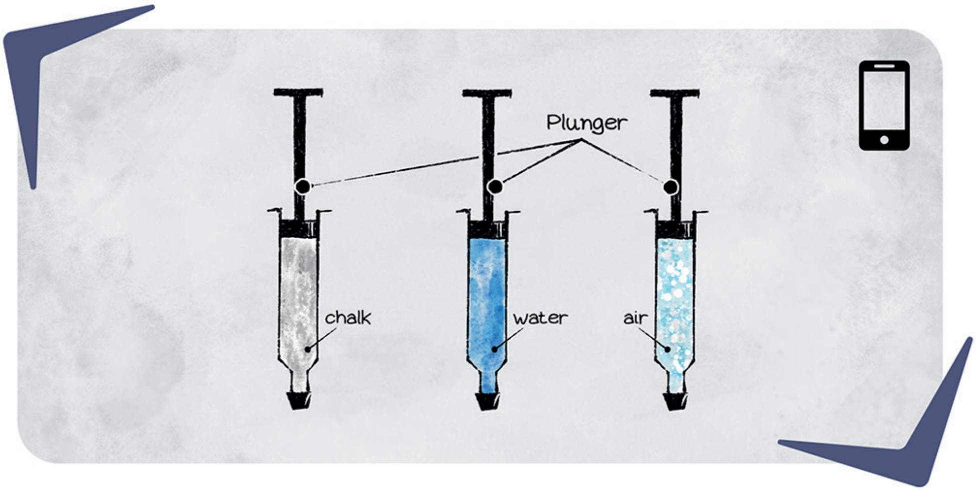
Your comments, question, observations.

Compressibility

Fluids can reduce their volume by applying pressure.
Watch the animation.
What did we observe?
Animation showed that air was compressible, water was hardly compressible, and chalk was not compressible



This is our example. Nobody wants to crash the car, that is why we control the air in pistons. But why? When the brake pedal is pressed a piston moves in a cylinder forcing a brake fluid through the brake lines to the brake callipers. While brake fluid will not compress, air does, and the reaction of braking system is slower. It is because at first, we need to compress the air and after that we are also pressing the fluid and make brakes work.



Do you know any application of compressibility in daily life? Put down your answers.

Is there any other common property for liquids and gasses we can test?

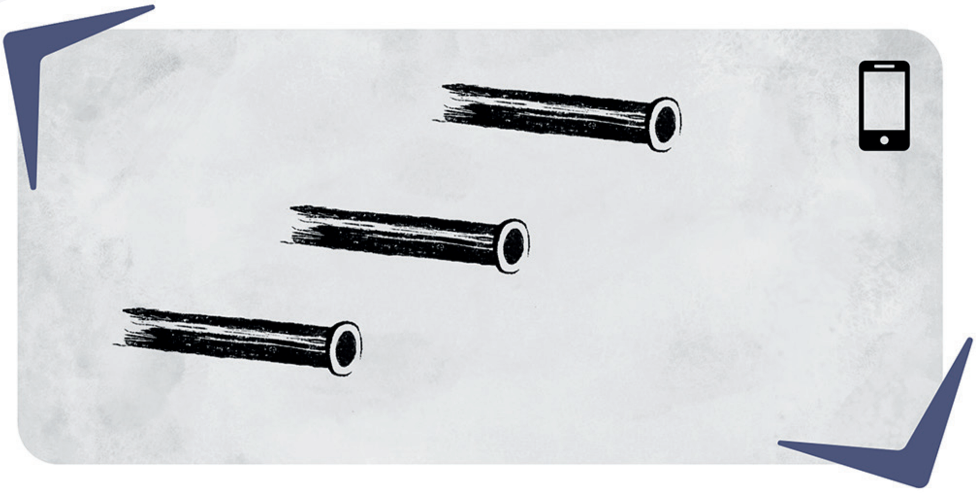
Let us take four different liquids: water, syrup, oil, and honey for example. Liquids can flow, but honey flows slower than water. Why? It has to do with viscosity. Follow me with the experiment.

Materials needed: glass balls, glass or plastic measuring cylinder for each liquid, stopwatch.
Procedure:

1. Fill cylinders with different liquids to the same height.
2. Drop the glass ball just above the upper level of liquid and your classmate measures the time till the glass ball reaches the bottom.
3. Fill in the data in the following table.
4. Heat the liquids in a water bath to a temperature at least 30 degrees higher and repeat the measurements for different temperature.

Liquid	Time in seconds for temperature 1	Time in seconds for temperature 2
Water		
Oil		
Honey		
Syrup		

5. Compare the results and make conclusions.

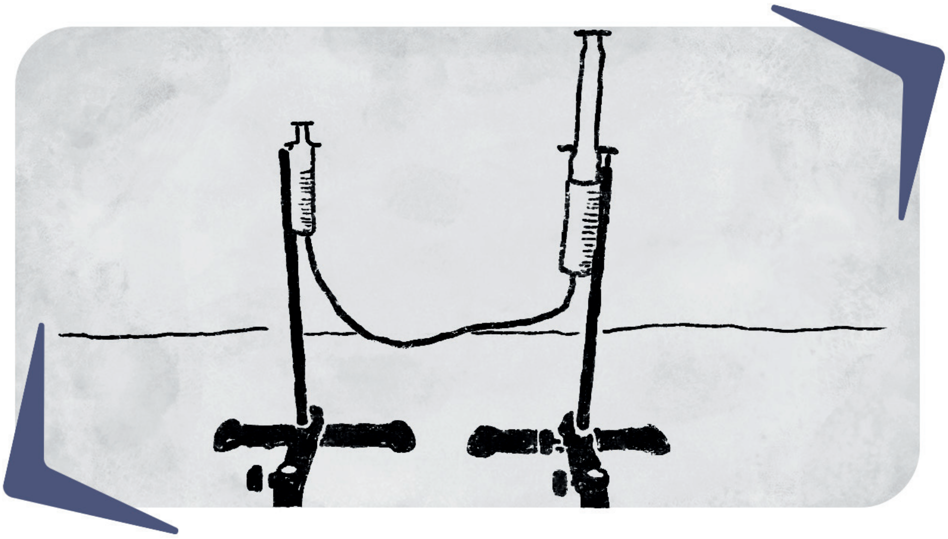


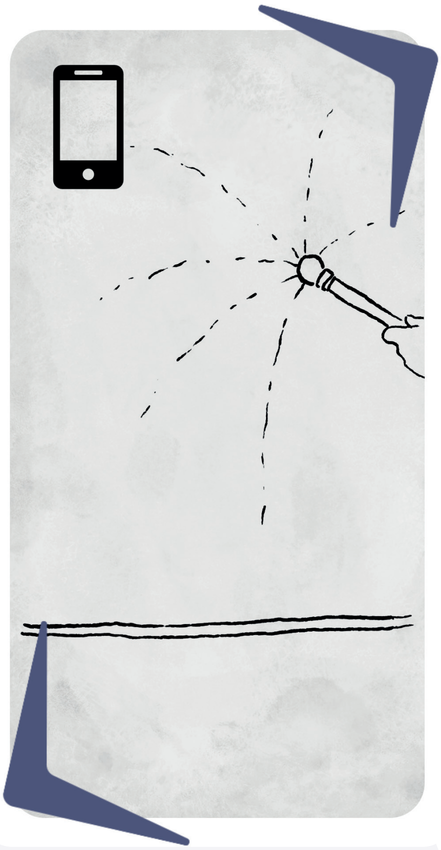
Hydrostatics - Pascal's Law

If the pressure at one point in a closed container increases, the pressure in the entire fluid also increases and the fluid pushes in all directions, perpendicular to the walls of the container.

There is a device called Pascal´s Hedgehog used for the demonstration of Pascal´s Law. There are many videos available on the Internet, find the one you like the most.

Your comments, question, observations.





What we observed was that when we fill the Pascal's Hedgehog with water, we will notice that pressing the piston causes the water to shoot out in all directions.

Are you a competitive person? Let us have a race between two different size syringes. Both syringes will be partially filled with water and then joined together. We will test the strengths by pushing on the pistons.

Which one will win the press wrestling? Small or big one?

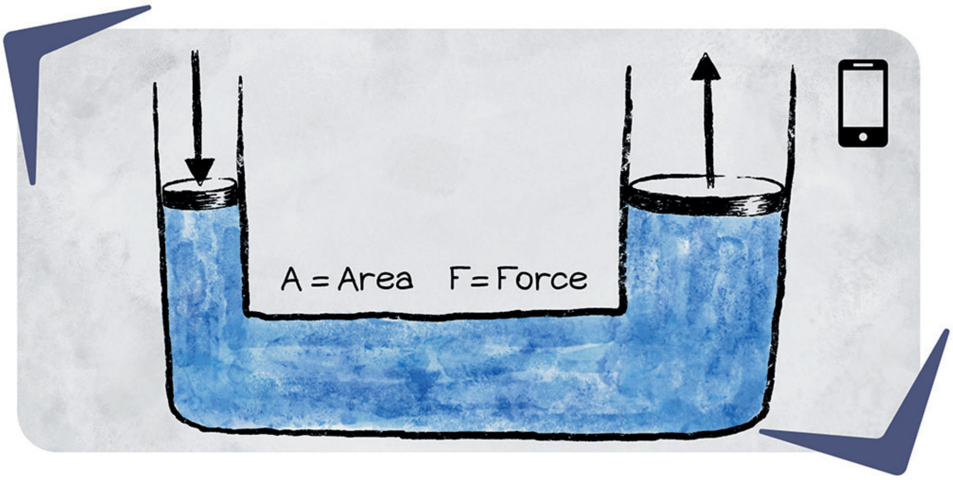
Where were two hands needed for the bigger or smaller syringe?

Your comments, question, observations.

When it comes to remembering the above results, we usually ask: How does this help? Try this: take two nuts. When you press one, you cannot break it, but when you hold two and you press, they touch on a small area, and you have a chance to break them. Or in winter when we want to walk on the snow, we enlarge the area of shoes so that we do not get bogged down. Or on the dance floor it is painful if a girl with stiletto heels steps on your toes. Pictures will help you to imagine and think.

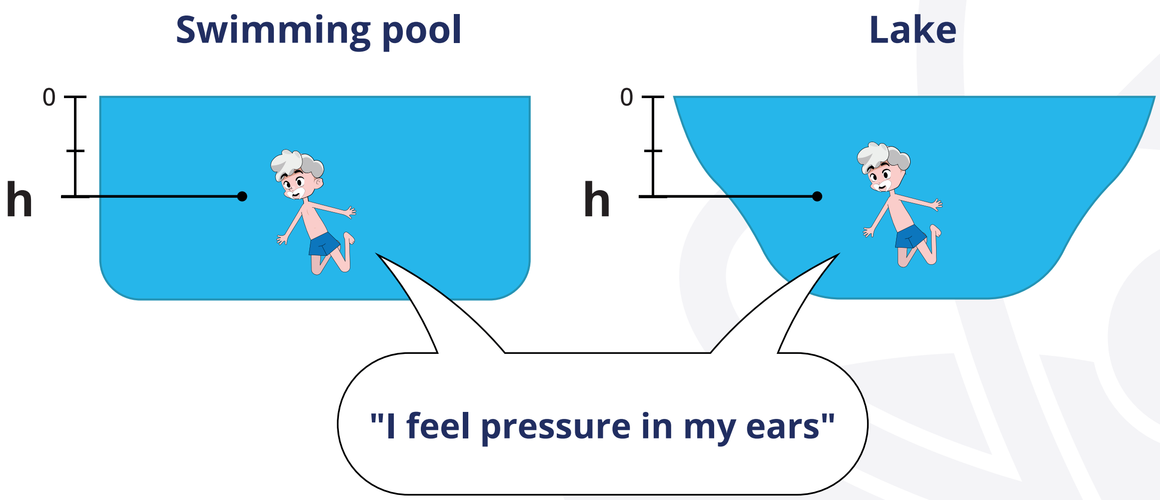
Another important application of the Pascal's law is Hydraulic jack. Remember: pressure is force applied on certain area; the total force exerted on a body containing a fluid is proportional to the surface area exposed to the fluid.

Your comments, question, observations.



Leakproof pistons are fitted in two arms of a U-shaped vessel with different cross sections, and the pressure on both pistons remains the same. If the area of the large piston is 100 times that of the small piston, the total force on the large piston is 100 times that on the small one. The work done (force × distance), however, remains the same in the absence of friction. Thus, the small piston must be pushed down 100 times farther than the large piston rises.

During the hot summer Arphy went swimming and diving in a swimming pool and a lake. I feel pressure in my ear drums. How can I calculate it? We can calculate the pressure in a container using a formula we already know.

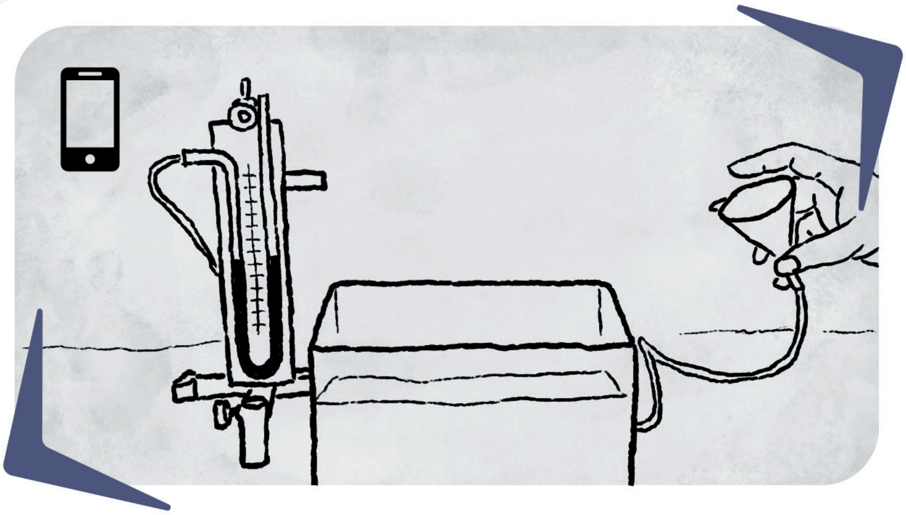


It is clear now, that it does not matter whether Arphy is in a pool, or a lake supposing he is at the same depth. Watch the video with the funnel in the water tank changing direction at the same depth.

Your comments, question, observations.

$$p = \frac{F}{A} = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho A h g}{A} = \rho g h$$

Where: *m* is the weight of water in container, *ρ* is the density of water, *A* is the area of the bottom of container, *h* is the height of container, *g* is the gravity acceleration.



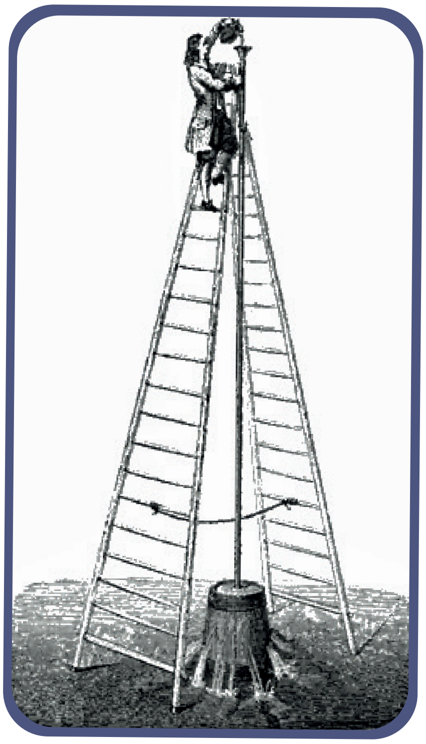


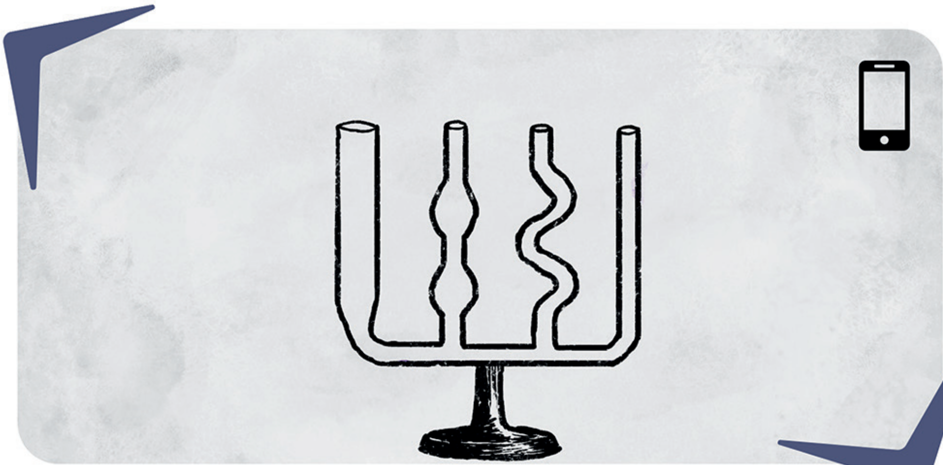
Fig. 34

It is said that Pascal performed the consequence of the formula $p=\rho gh$ in a very interesting way. The idea is presented in the picture.

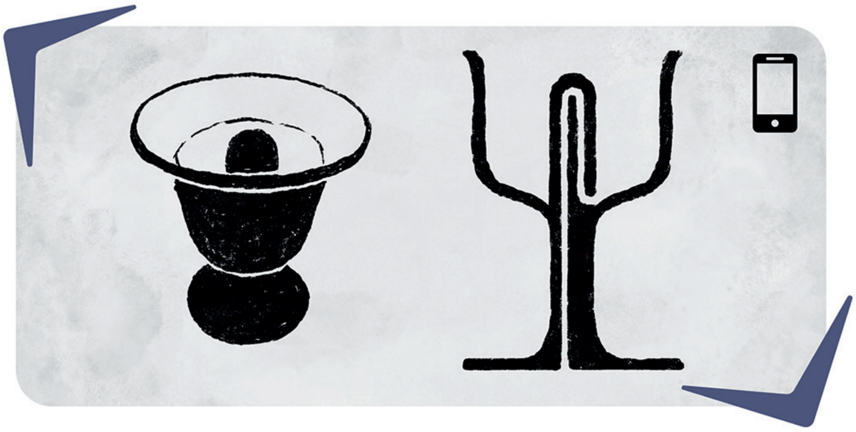
There is one interesting consequence of hydrostatic pressure, called hydrostatic paradox. Paradox is called something that is contradictory, but in this case, there is nothing contradictory, it only needs more thinking. To explain what is behind hydrostatic paradox we need to start with Pascal's law of communicating vessels

Communicating vessels are a set of connected containers filled with homogeneous fluid. When the liquid settles, it balances out to the same level in all the containers regardless of the shape and volume of the containers. The hydrostatic pressure is the same in all containers, because as we can see in the formula, it depends on the height, gravity acceleration and the density of fluid, so the pressure does not depend on the shape of the container, and it is called hydrostatic paradox. Back to Pascal's experiment, he needed only small amount of water, but the height was important. When dealing with hydrostatic pressure it is also necessary to keep in mind that in the same depth it is the same in all directions.

This hydrostatic experiment was allegedly performed by Blaise Pascal in 1646. He inserted a long vertical tube into a solid barrel filled with water. He poured only a glass of water into the tube and barrel exploded. The experiment is mentioned nowhere in Pascal's preserved works. It was attributed to him by 19th – century French authors, they called it „crève-tonneau de Pascal." There are many videos repeating this experiment on youtube. Find the one which will help you to answer the question. Why did the barrel break?



Your comments, question, observations.



Your comments, question, observations.

So far, we have only applied Pascal's Law in the case of liquids, mainly water. Does the same principle apply to gasses? It is important to know, because we live in an ocean – at the bottom of the ocean of air. Since we live in this ocean from our birth on, we don't notice its presence until some change occurs, such as when we go hiking in the mountains.

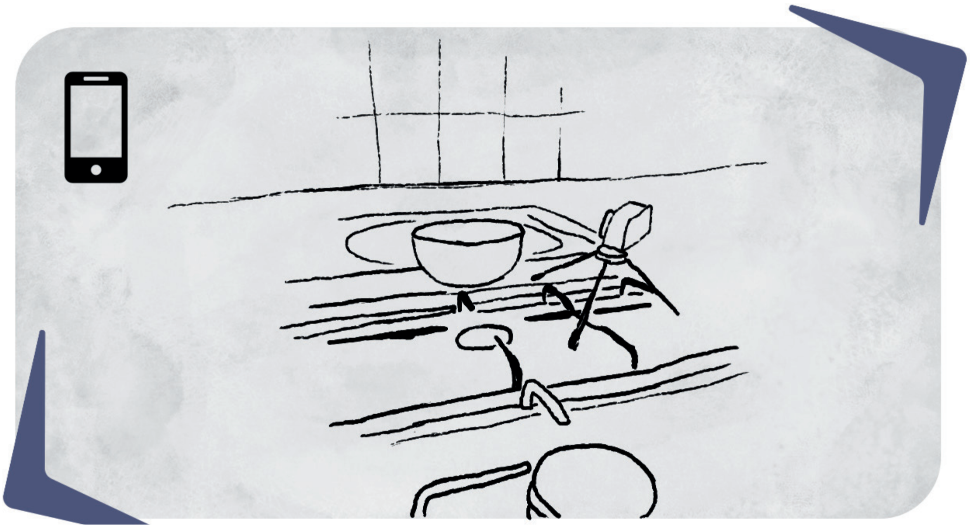
Scientists always liked to play. Many gadgets were invented to amaze colleagues or students. Following is one that amazes and makes fun since Pythagoras time. Archimedes taught his students, with the cup of justice, not to be greedy when drinking wine. Behind this prank is one of the most famous ancient Greek philosopher and mathematician, Pythagoras. He is the mastermind behind the Pythagorean Cup or cup of justice. The cup is based on a genius design, and still tricks people to this very day. Do you want to know how it works? Watch the animation.

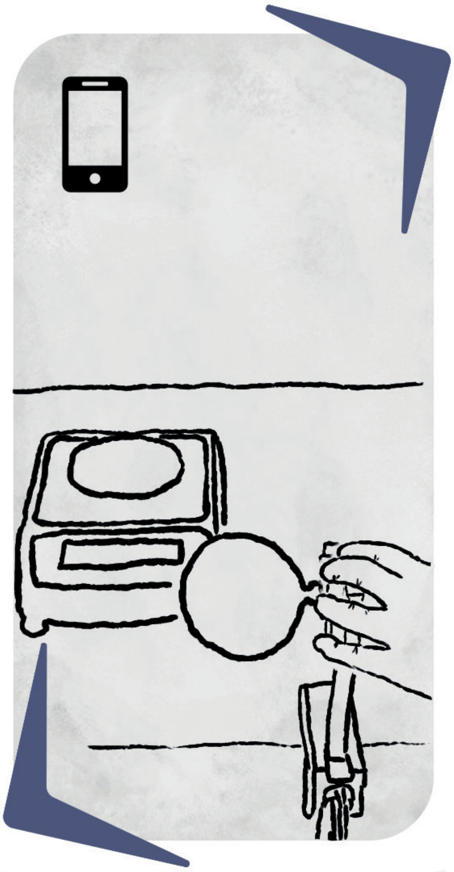
Climbing up to high mountain areas, the pressure of the air decreases, the body needs to cope with the lack of oxygen in our lungs. There is an increase in breathing and heart rate to as much as double, even while resting. Another way to feel that air has higher density on Earth surface is when the airplane lands, you feel like the airplane lands on some cushion. The air is denser close to surface.

The cause of hydrostatic pressure was the weight of water, but we do not feel the weight of air. Does air weigh? How much is it?

Now, we can agree that air has weight. Let us do some calculation and see what the numbers reveal. Having a tube with an area of 1cm^2 and a height equal to the atmosphere above us, the mass of air in it would be 1 kg. It does not seem that much; it is the weight of 10N per cm^2, and that is 100 000N per m^2. (Mass of a hippopotamus is up to 3000 kg, which means 30 000 N weight). This value is known as atmospheric pressure, which is often mentioned in weather forecasts. Some more numbers, the area of hand is approximately 150 cm^2 and that means 150 kg of air on hand. The force exerted on hand, the weight of air is 1 500N. But there is the same force acting on the underside of the hand and so we live comfortably in this ocean of air.

Watch crashing the can caused by atmospheric pressure.





Your comments, question, observations.

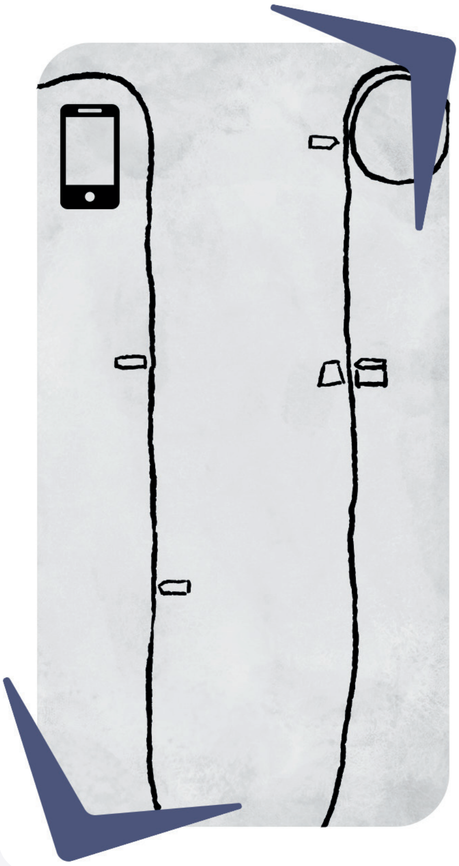
By watching the can crash, you can observe the strength of the atmospheric pressure. Do you have any idea, why does it happen? Or you may look for an answer.

Your comments, question, observations.

Hydrostatic pressure, pressure difference and snorkeling.

We need 2-3 m long transparent (plastic) hose. We will make the form of a U-tube and fill it partially with water. We blow the air in one arm by mouth, overpressure in the mouth can be determined from the difference of heights *h* levels in both vertical tubes. $p = \rho g h$, where ρ is the water density, g is gravity acceleration. Will there be a difference if we suck the air?

> [1] *What did the video help us with? Not many of us experience scuba diving, but snorkeling is different. According to the video the hight of the column when breath out was 870 mm and for breath in it was 820 mm. Will we be able to snorkel comfortably in such depth? With some calculation (https://arphymedes.eu/) we will find out that the force on the area of the chest will be the same like 52 kg person sitting on the chest. And the pressure is from all sides, remember. That is why the length of the snorkel is only about 60 cm.*



Now something from daily life, to help you understand the pressure difference. **When we measure the pressure of gasses, like that of the air in car tires, we usually make this measurement relative to the normal air pressure. Pressure difference means the difference between the pressure in the tire and atmospheric pressure**.

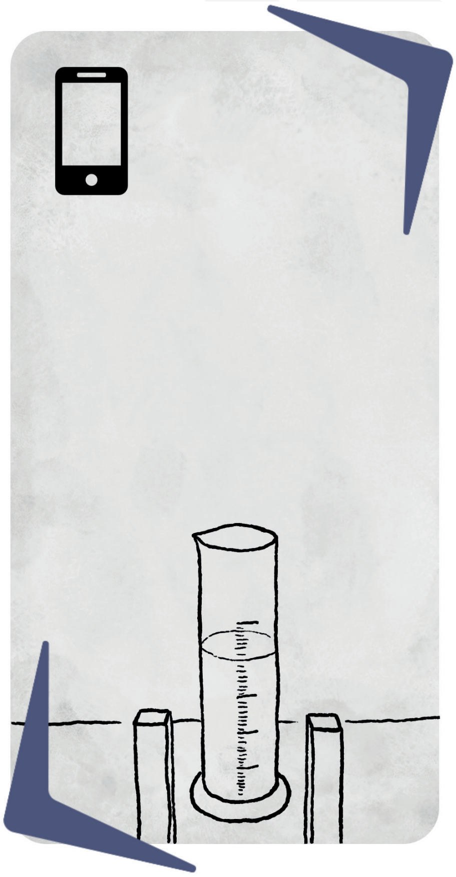
Watching films about underwater world is quite popular. We see divers with pressured air tanks diving deep under the water. Why do they need it? When scuba diving the hydrostatic pressure is so big, that to be able to breathe we need pressured air in the tank with us and so we counter the hydrostatic pressure of the water. Even in ten meters under the water the hydrostatic pressure would be 100 000 Pa, which means the hydrostatic pressure force on our chest (supposing 30cmx20cm being the area of chest) 6 000N, it is as 600 kg person would sit on your chest. But there is the same pressure in all directions, on our chest, back, both sides. We will be squeezed. (All calculations are done for fresh **water**[1]). Can you say, what is more difficult, to breathe in, or out?
It is important to remember that we have only calculated the water pressure at the given depth. Additionally, there is the air pressure acting on the water surface.

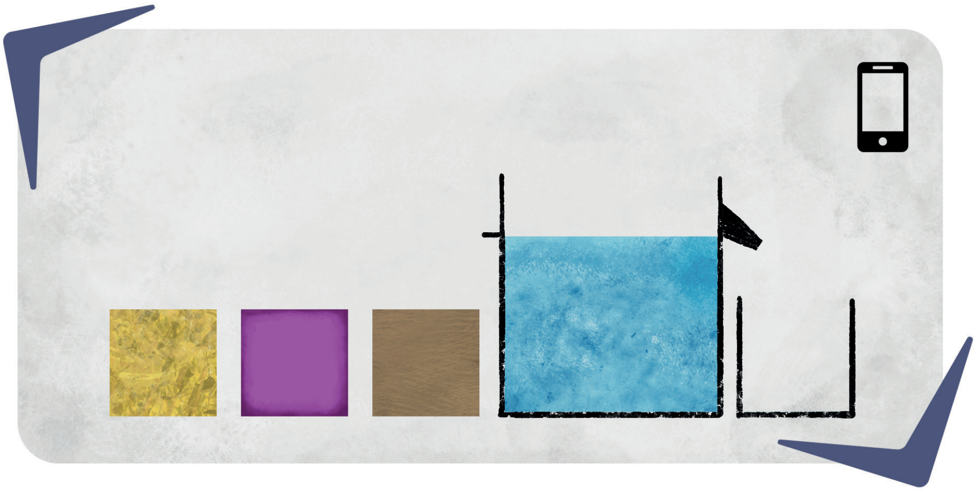
Hydrostatics - Archimedes Principle

Please stop, do not hurry to fill the bath! Just start with us the Archimedes Quest. We will take the same journey as Archimedes. We will observe, describe, experiment, put the results into mathematical language and we will play and have fun. At the beginning there was a bathtub filled to the brim with water. When Archimedes entered the bath, some amount of water overflowed. What will happen when Archimedes would be bigger or smaller? Will the amount of overflowed water be the same? We do not have Archimedes to test it, but we can make experiments with objects with different shape, volume, and density. All these must be tested and do not forget, we can always change only one parameter, and water must be filled up to the brim. Experiment 1 will test objects of same size, shape, but different density. Colours assigned to the experiment number match with the colours in the Archimedes Quest.

What was the amount of the overflowed water? Was there a difference? How did it depend on the "void left by the objects"?

| [1] *The amount of overflowed water always filled the "void left by object".*





Your comments, question, observations.

Probably each of us know the feeling of lightness when immersed in water. Experiment 2 will test whether there is a difference between the weight of objects when measured in air and water.

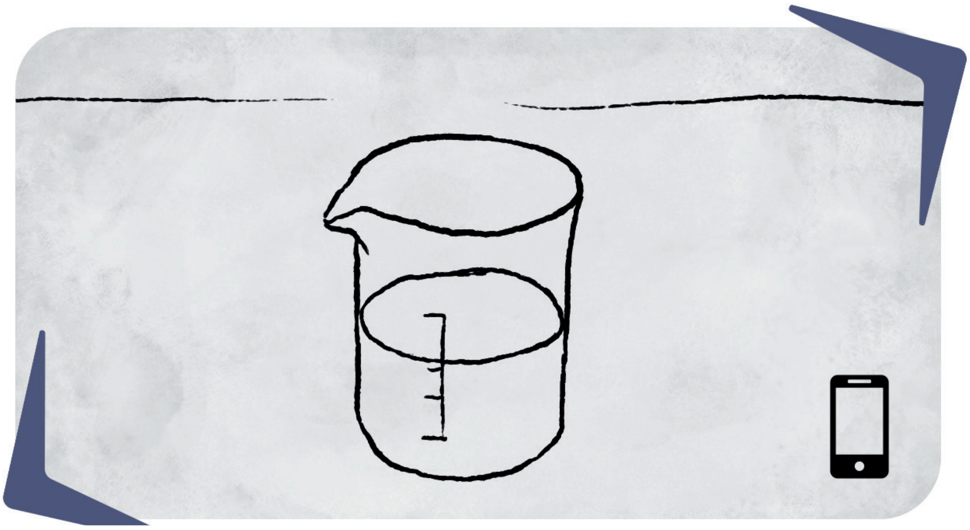
> [2] *All objects seem to weigh less in water. We know from the chapter about adding forces, that difference between 2 forces is force again and we will call this one Buoyant force. G(in air) - G(in water) = Buoyant force*

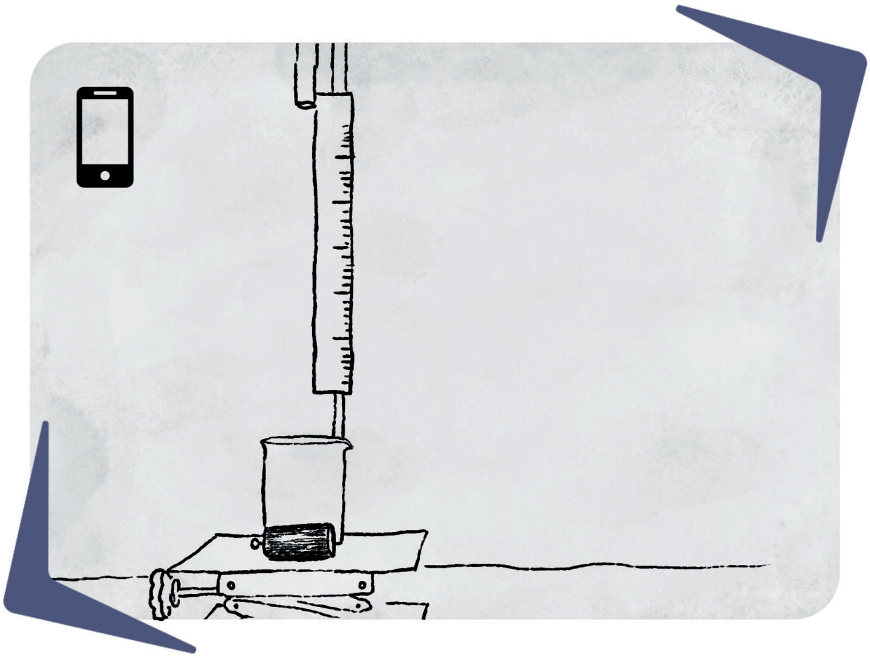
Your comments, question, observations.

You may have device called Archimedes double cyllinder at school. If not watch Experiment 3.

The summary so far allows us to define: Archimedes' principle states that the upward Buoyant force that is exerted on a body immersed in a fluid, whether fully or partially, is equal to the weight of the fluid with the same volume as the immersed part of a body.

Your comments, question, observations.





The quest follows, we need to find out what Buoyant force depends on. Experiment 4 is designed to test whether Buoyant force depends on the amount of the water in the container. Is there a difference when we weigh the same object in the same container with different levels of water?

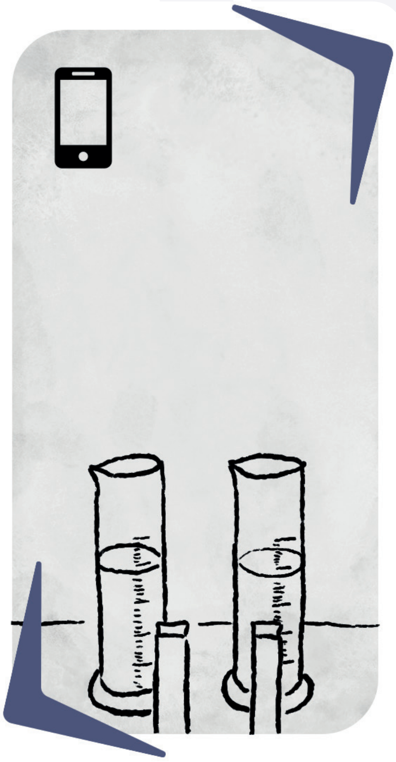
> [3] *If we put the same object into different levels of water, there is no difference in the Buoyant Force.*

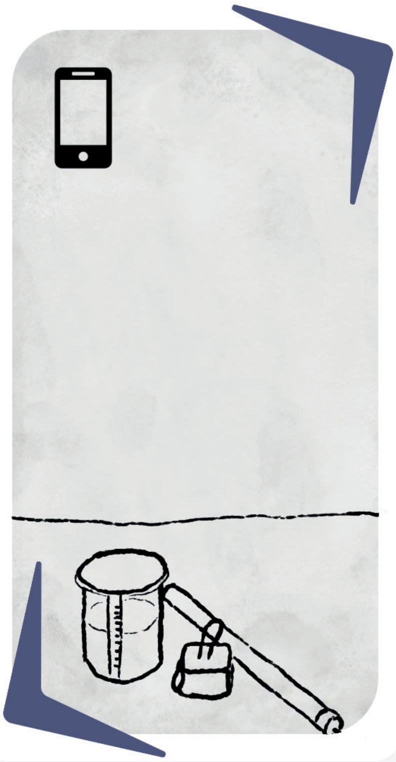
Your comments, question, observations.

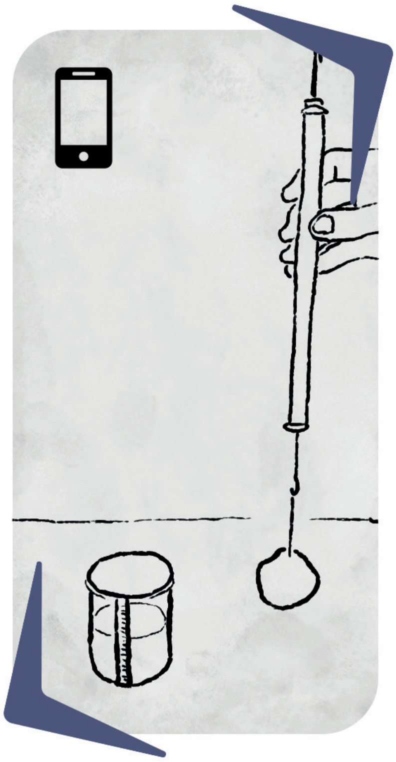
What will happen if we use objects with different shape, but with the same volume and from the same material. Experiment 5.

> [4] *If we put the objects from the same material and volume, but different shape there is no difference in the Buoyant Force.*

Your comments, question, observations.







A lot of us experience, that it is easier to swim in the sea than in the lake. Testing whether there is a difference when there are different liquids (like for example salt and lake water), is done in Experiment 6. As there is no difference in colour in salt and lake water, the denser liquid is coloured.

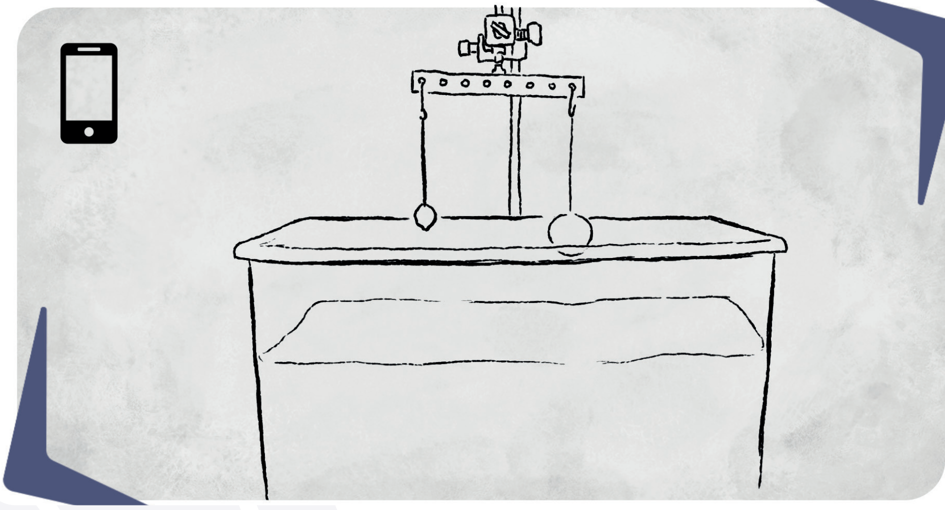
[5] *If we put the same objects into the lake and sea water, there is difference in the Buoyant Force. The Buoyant force increases with increased density of* ***liquid****.*

Your comments, question, observations.

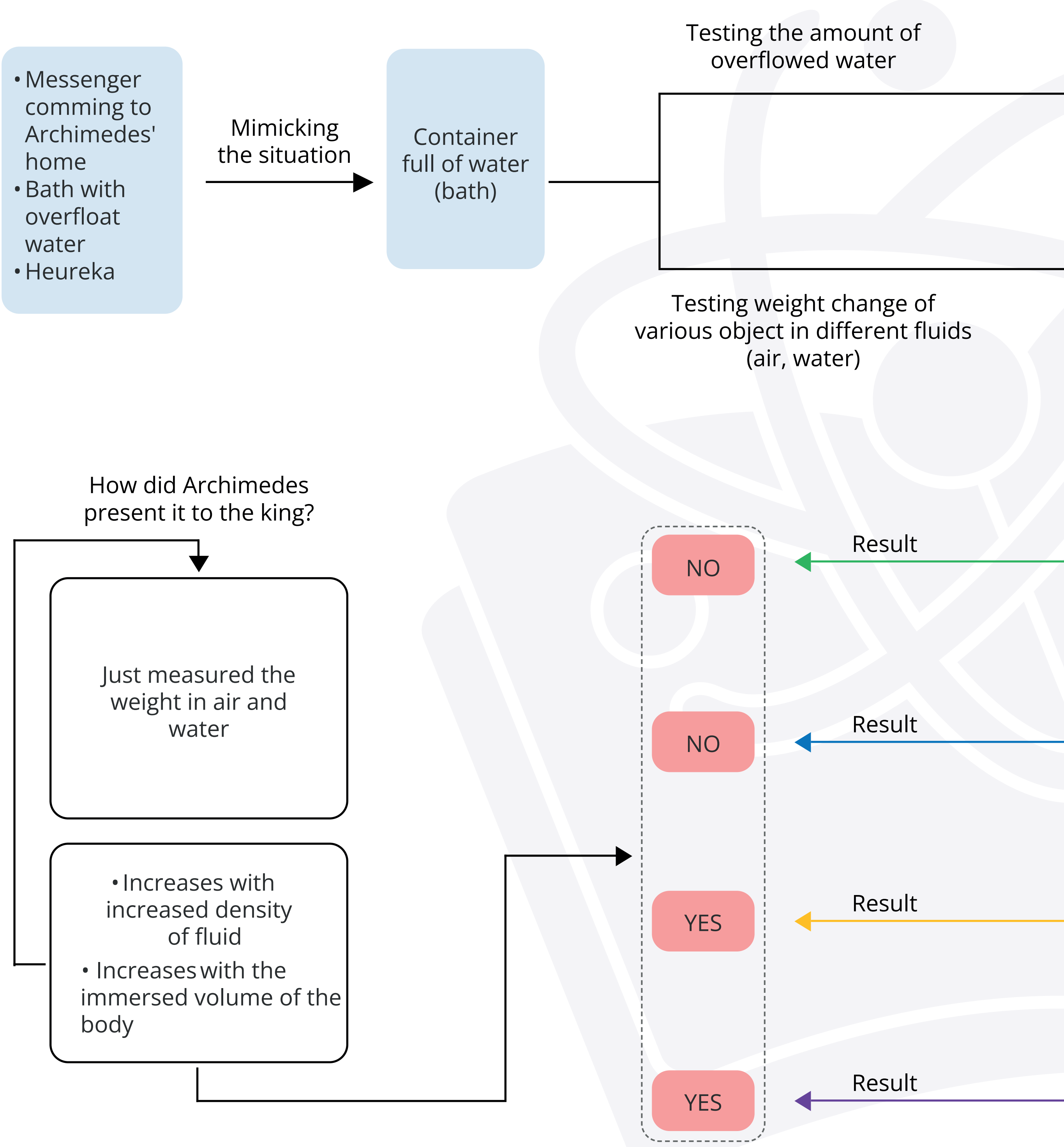
Now we can test if Buoyant Force depends on the material object is made from. Experiment 7. Can you put together all the results?

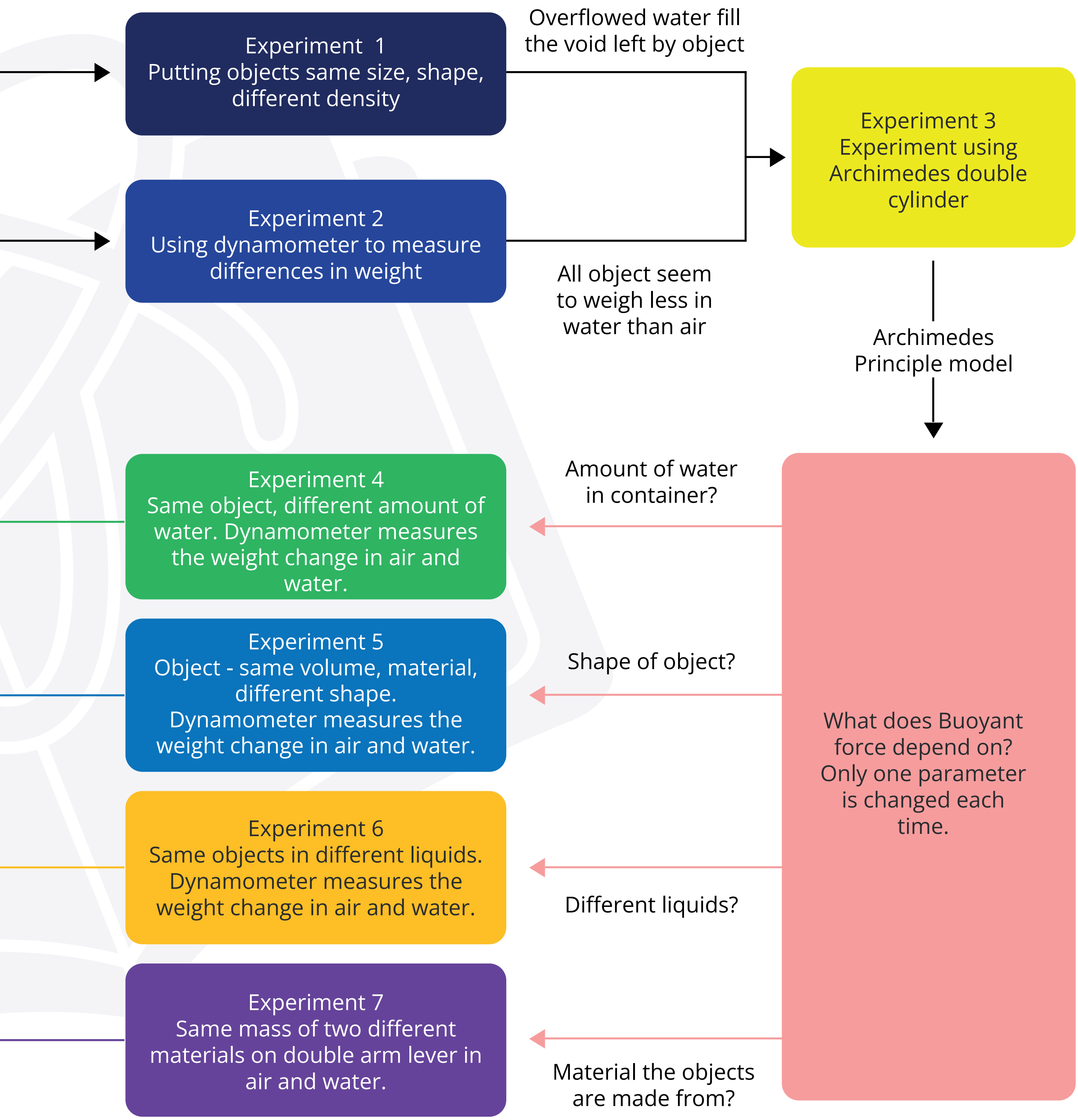
> [6] *Summary: The Buoyant force depends on: The density of the fluid, the volume of immersed part of the body. Now, we have everything. On one side of the double side lever put the crown and on the other side gold with the same weight. If the goldsmith used some amount of silver, what would happen? The object from denser material will have less volume. (Smaller sphere).*

Your comments, question, observations.



Archimedes Quest




Experiment 1
Putting objects same size, shape, different density
Experiment 2
Using dynamometer to measure differences in weight
Overflowed water fill the void left by object
All object seem to weigh less in water than air
Experiment 3
Experiment using Archimedes double cylinder
Archimedes Principle model
What does Buoyant force depend on? Only one parameter is changed each time.
Amount of water in container?
Experiment 4
Same object, different amount of water. Dynamometer measures the weight change in air and water.
Shape of object?
Experiment 5
Object - same volume, material, different shape. Dynamometer measures the weight change in air and water.
Different liquids?
Experiment 6
Same objects in different liquids. Dynamometer measures the weight change in air and water.
Material the objects are made from?
Experiment 7
Same mass of two different materials on double arm lever in air and water.

Hydrodynamics

Hydrodynamics is a bit advanced physics, but since we often encounter it in everyday life, we will try to approach it through experiments.

The basic laws are laws of conservation of mass and energy. The law of mass conservation states that when fluid flows through a full pipe, the volume of fluid entering the pipe must be equal to the volume of the fluid leaving the pipe, even if the diameter of the pipe varies. Good news, otherwise, we could have a problem with tap water. The pipes are thinner than the plumbing, but thanks to the law of conservation of mass, as much water flows in, as much flows out.

Conservation of energy is behind the following experiments. They have one thing in common they show the cases when the speed of a fluid increased and the pressure decreased. There are also other experiments your teacher can show and explain to you. **Experiment 1**.

Experiment 2 | Please be alert and aware while on all platforms. For your safety, please remember to **stay behind the yellow strip at all station platforms before boarding and after exiting the train**. There is Bernoulli's principle behind this sign on train stations. Can you explain it?

Experiment 1

When we blow between two cans, we create an area of low pressure (the faster the air moves, the lower the pressure).The relatively high-pressure air on the outer sides of the cans will rush over to fill the area of low pressure ("winds will blow from high to low"), effectively pushing the cans together. You can try it also by blowing between the two paper sheets, or two balloons.

Your comments, question, observations.

Experiment 2

When a fast-moving train rushes past the train platform, the velocity of air next to train increases, it means that air pressure decreases. Standing close to the train the air flowing from the higher-pressure area to the train can push the person towards the train and under its wheels. The same can happen if a fast-moving truck overtakes a cyclist from an insufficient distance. That is why it is important to have a separate road lane for cyclists.

Your comments, question, observations.

Experiment 3 | There is an annoying experience you might have when taking a shower in shower cabin with curtain. When having a shower, the curtain is pulled towards you and it gets wet. Sometimes it even sticks to you.

Example from life.
In Boston in 1972 wind blowing around skyscraper with high speed caused the windows popped out. Find out more on the Internet.

Experiment 3
If you like to use bigger amount of water and even massage your body with it, the water has a high speed. According to Bernoulli's equation, the higher the speed, the lower the pressure and it means the curtain is pulled towards the stream of water and gets wet.

Your comments, question, observations.

Boston example explained.
In Boston in November 1972 the glass was literally falling when a new skyscraper encountered the seasonal winds. The unfinished 60-storey John Hancock Tower had more than 10 000 window units, each with 12 square metres of glass. When the wind blew the panes started shattering, and broken fragments rained down. The explanation is again in Bernoulli's equation, the high speed of the wind caused the low pressure outside the building and so the glass moved to the area of lower pressure and broken fragments were falling on the street.

Your comments, question, observations.

On the last page you can find two 3D models mentioned in this chapter.
You will find the explanation in the additional chapter Project Proposals and Technical Applications.

3D model of an airplane.

3D model of a submarine.