MECHANICS: KINEMATICS

ARPHYMEDES

Nicole Oresme (c. 1320–1325 – 1382)

Fig. 8

Philosopher, scientists, Bishop of Lisieux, a translator, a counsellor of King Charles V

Nicole Oresme was a French philosopher of the later Middle Ages. He wrote about many different subjects like economics, math, physics, astrology, and theology. He was also Bishop of Lisieux, a translator, a counsellor of King Charles V of France, and one of the most original thinkers of 14th-century Europe. Oresme had his own ideas about how things move, as we can find it in his book called Physics. He didn't agree with what other famous thinkers like Aristotle and Averroes said about motion. He believed that objects don't just have motion naturally, but that something has to make them move. He had his own unique way of thinking about it.

René Descartes (1596–1650)

The first to come up with modern ideas about how things move and why they do.

René Descartes is known for being one of the important people in philosophy and also in physics. But for a long time, people didn't realize how much he contributed to physics. Descartes was the first to come up with modern ideas about how things move and why they do. He also had a theory about how planets move, which was very popular back then. He had a habit of staying in bed late in the morning, but he had to change that when the Queen of Sweden invited him to teach her. The Queen wanted lessons at 5 in the morning, so Descartes had to wake up early and walk in the cold winter. Unfortunately, he got very sick with pneumonia and passed away.

Fig. 9

André-Marie Ampère (1775 – 1836)

Fig. 10

Clever to be a member of respected scientific institutions without any formal education.

André-Marie Ampère was a French physicist and mathematician who was one of the founders of the science of classical electromagnetism, which is all about electricity and magnetism working together. He used his knowledge in the design of several technical devices, such as the solenoid and the electrical telegraph. Even though he did not have any formal education, Ampere was a member of the French Academy of Sciences, and professor at the École polytechnique, and the Collège de France. Ampère also came up with a word, "kinematic," which means movement or motion. He made this word using Greek words κίνημα kinema ("movement, motion"), and κινε ν kinein ("to move").

On this page you can find some proposals for projects that you can make at school or at home. There are even more on the accompanying webpage.

Project proposals:

1. Try to find out how you can measure your reaction time.

2. Make your own story of somebody's day. Find out necessary information to be able to make the plots speed – time and distance time.

3. Find some application into your mobile phone that tracks the number of steps taken in a given time, the total distance traveled, even the number of calories burned and the average speed for the day. Use it for a day and then answer the following questions
   - How many steps did you take during the whole day?
   - Try to find out, how you can calculate the average length of your step.
   - How far did you walk in the whole day?
   - What was the distance you made between 7.30 and 9.a.m.
   - What was your average speed for the whole day?
   - Make plot distance – time for the chosen time interval.
   - Prepare one task for your classmates to calculate, based on the data from application.

Technical applications

1. Doppler effect – explanation and application in science.

Mechanics is the science of things in motion.

Fig. 11

Fig. 14

Fig. 12

Fig. 13

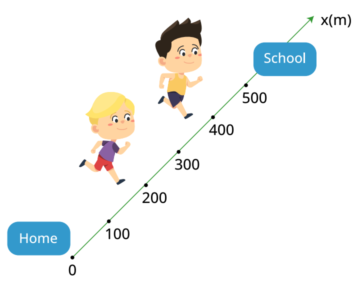


Fig. 16

Fig. 15

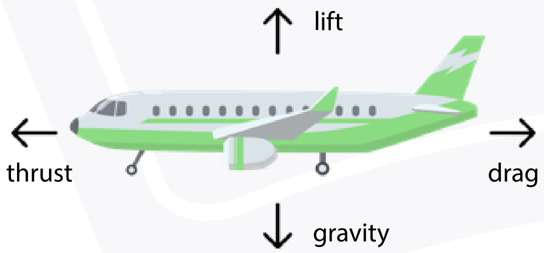
Looking at the moving objects, there are two main questions:

How? – Kinematics is describing the motion of objects using words, diagrams, numbers, graphs, and equations.

Fig. 18-19

Describing the movement is not that easy. Let us start with static situation. **Try task 1.**

Why? – Dynamics relates to forces and their effects on motion.



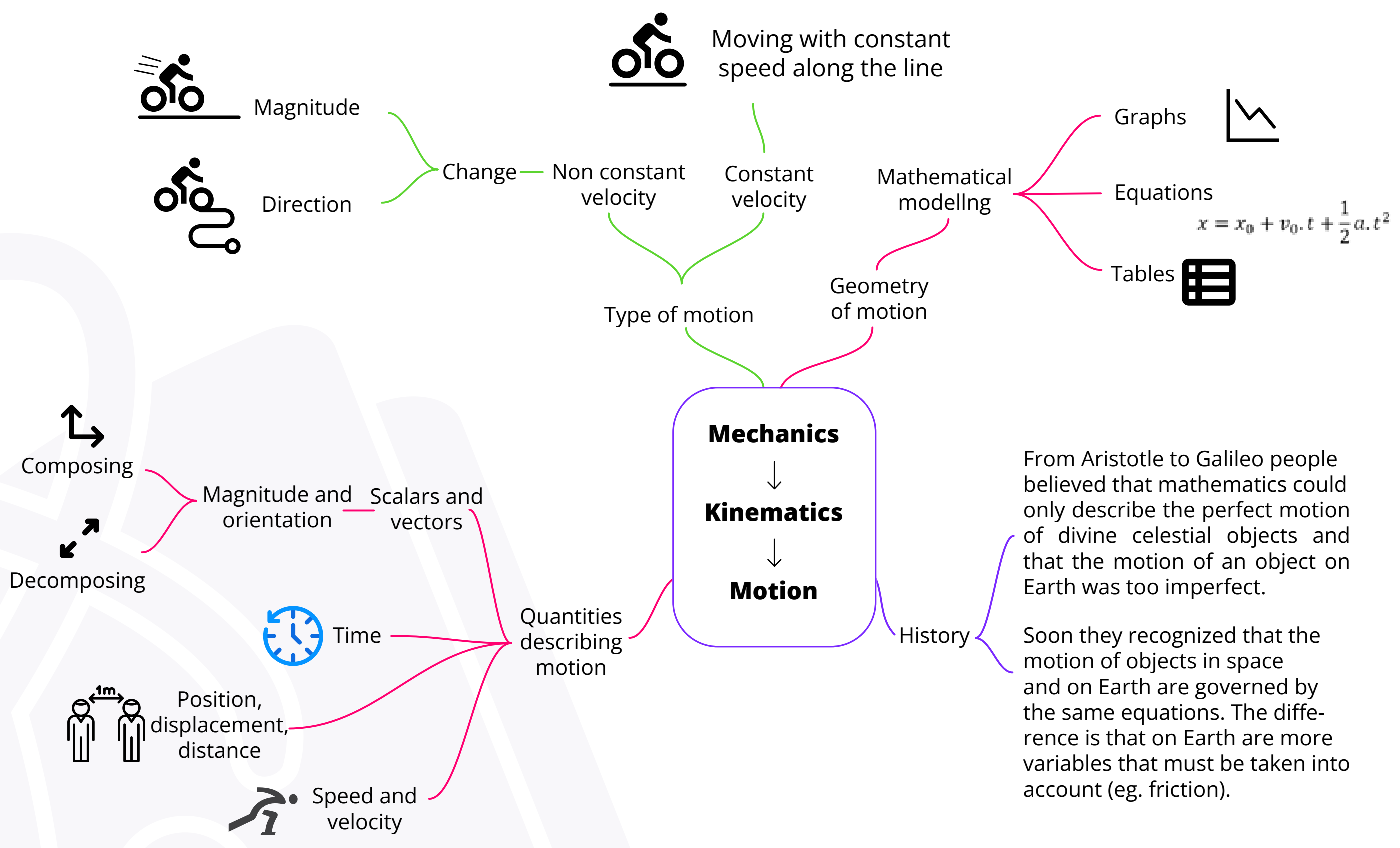
Task 1. Give your classmate a piece of paper. Only you will have the picture of the classroom. Describe the position of the children so that they will be able to mark it on their papers. Then do the same with the checkered paper and decide that you will count from the down left corner

Fig. 17

The mind map shows the many things (terms) connected with motion. We will proceed step by step and it is good to return to the mind map with each new information.

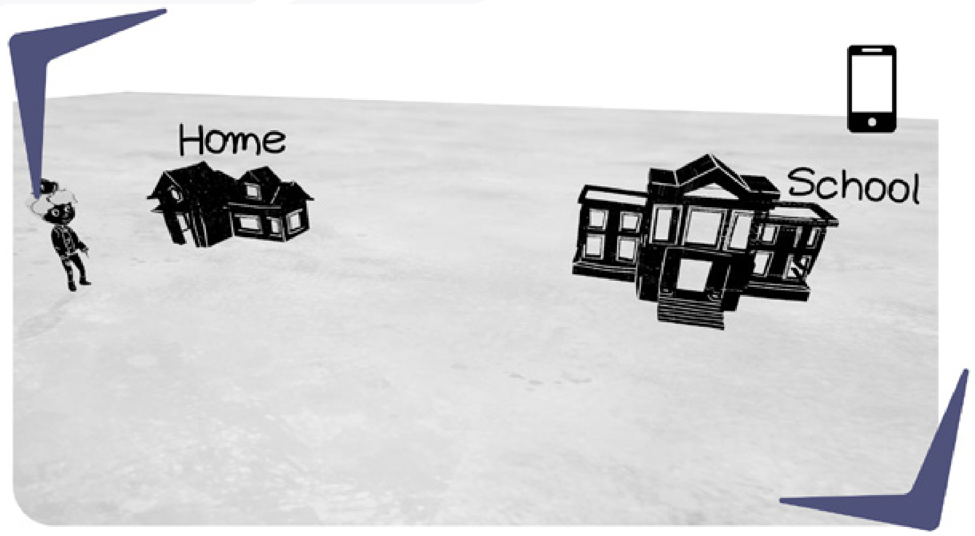
Our goal is to describe motion.
How do we identify motion? What does it mean that a body moves?
What physical quantities are important to describe motion?
What are the ways in which we can describe the motion?
What are the simplifications we can make?
What types of motion are known?

To study properly how a body is moving we must know few other things.
Motion is mathematically described in terms of displacement, distance, velocity, acceleration, speed, and time.
The motion of a body is observed by attaching a frame of reference to an observer and measuring the change in position of the body relative to that frame with change in time. That is why checkered paper in task 1 helped under the condition that all of you started counting from the same beginning. Right, left, up, and down were the only instructions you needed.



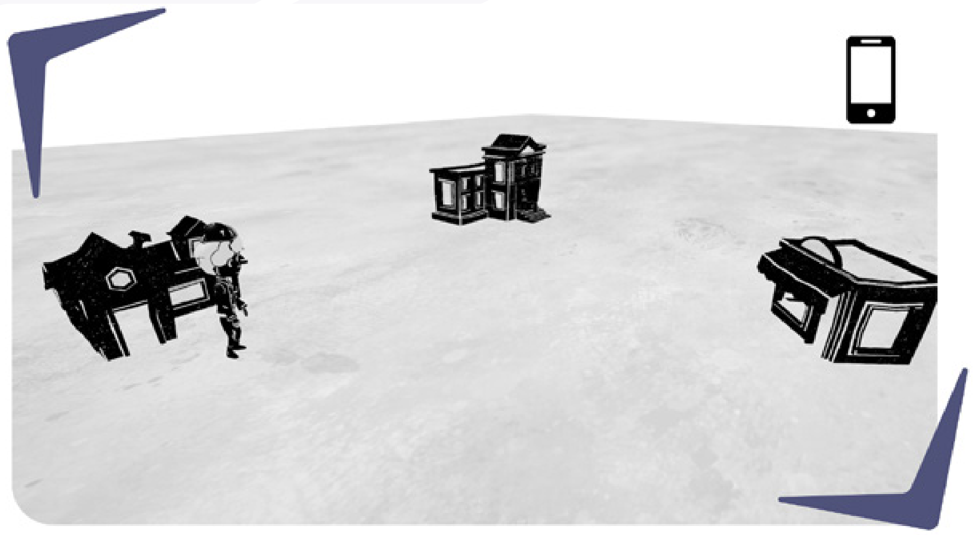
▸ Position, displacement, distance.

Arphy went from home to school. Watch the animation.
If we want to describe what we see, the only thing we can say is that Arphy went from home to school. To find out more something must be added. Any idea?



Your comments, question, observations.

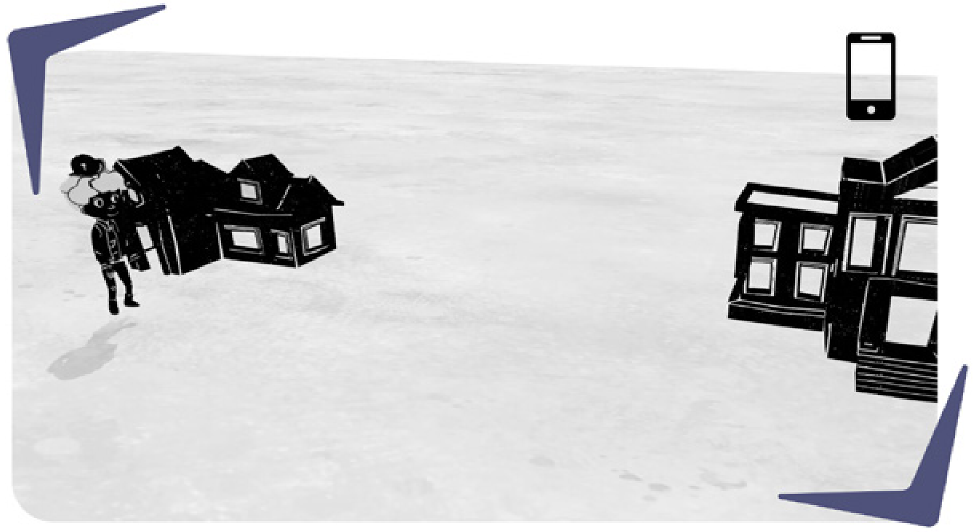
Let us add another information into the animation.
Now, we can see the **position** of Arphy. He is located at 100m, 200m, 300m, 400m, 500m and finally at 600m he is at school. These 600 metres represent the length of his path. **We call it distance.**



Do you go to school in a straight line? We mostly do not go that way, just like Arphy in our example. He made a stop at the shop. Watch the animation. It will help us understand another term- **displacement**.
This time he walked a longer distance, with the change in distance being 390m+455m, totaling 845. However, when considering only about the shortest path between home and school it remained the same - 600m and this is called **displacement**. (Arphy's path started at home and ended up at school.)

Arphy was thoughtful going to school and this is how it turned out. Watch the animation.

Arphy made the whole way to school and then he turned around and came back home. He walked the entire path to school and the return journey. Regardless of the direction chosen, the distance simply get added together. **Displacement** means that you sum the whole path, but you take care of direction. Going to school is one direction (+), while returning home is viewed as a negative direction (-). Arphy started at home and ended up at the same **position**, at home. **Displacement, which is the path between the final and starting (initial) point was zero**.
Displacement and distance are measured in the same SI unit of length– metre.



So far, we have discussed the concepts of position, displacement, and distance, but Arphy could have been walking to school the whole day long. So let us add time to our description. Knowing where and when will give us more information. Watch the animation. By having knowledge of both the location and time, we can calculate the speed. **Speed** (*v*) is the rate at which an object changes its location.

$$v = \frac{100\text{m} - 0\text{m}}{t_{100\text{m}} - t_{0\text{m}}} = \frac{100\text{m}}{100\text{s} - 0\text{s}} = \frac{100\text{m}}{100\text{s}} = 1\,\frac{\text{m}}{\text{s}}$$

$$v = \frac{200\text{m} - 100\text{m}}{t_{200\text{m}} - t_{100\text{m}}} = \frac{100\text{m}}{200\text{s} - 100\text{s}} = \frac{100\text{m}}{100\text{s}} = 1\,\frac{\text{m}}{\text{s}}$$

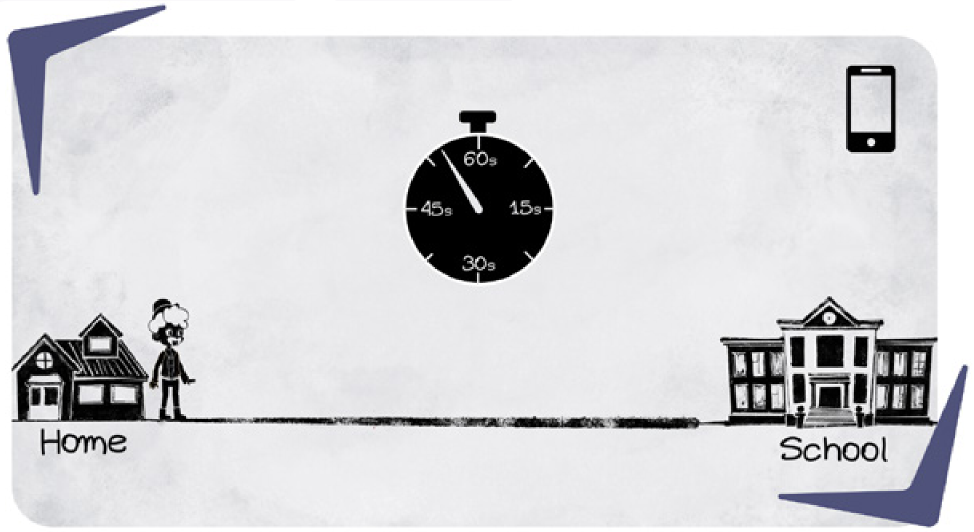
$$v = \frac{300\text{m} - 200\text{m}}{t_{300\text{m}} - t_{200\text{m}}} = \frac{100\text{m}}{300\text{s} - 200\text{s}} = \frac{100\text{m}}{100\text{s}} = 1\,\frac{\text{m}}{\text{s}}$$

$$v = \frac{400\text{m} - 300\text{m}}{t_{400\text{m}} - t_{300\text{m}}} = \frac{100\text{m}}{400\text{s} - 300\text{s}} = \frac{100\text{m}}{100\text{s}} = 1\,\frac{\text{m}}{\text{s}}$$

$$v = \frac{500\text{m} - 400\text{m}}{t_{500\text{m}} - t_{400\text{m}}} = \frac{100\text{m}}{500\text{s} - 400\text{s}} = \frac{100\text{m}}{100\text{s}} = 1\,\frac{\text{m}}{\text{s}}$$

$$v = \frac{600\text{m} - 500\text{m}}{t_{600\text{m}} - t_{500\text{m}}} = \frac{100\text{m}}{600\text{s} - 500\text{s}} = \frac{100\text{m}}{100\text{s}} = 1\,\frac{\text{m}}{\text{s}}$$

What do we know now about Arphy's movement? Arphy went from home to school. He was moving with constant speed 1 m/s. The situation can also be expressed in graphs, as we will do later.



If we sum up the previous pages: we calculated distance as the difference between the ending and initial position and the time interval as the difference between the ending time and the beginning time of a selected interval. (It is important to remember that difference is always ending – beginning. This might help you. For example, if you visit a hairdresser, your friends will compare your final state to your initial view, not the other way round).

Arphy was moving with constant speed and so the result was always 1m/s.

But in real life we mostly do not move with constant speed. Therefore, when we describe an object's speed, we often describe the average over a time interval. Average speed is the total distance traveled divided by the time during which the motion occurs.

$$v_{average} = \frac{total\ distance}{total\ time}$$

As Arphy was moving at the same speed, the average speed was the same.

$$v = \frac{600\text{m} - 0\text{m}}{t_{600\text{m}} - t_{0\text{m}}} = \frac{600\text{m}}{600\text{s} - 0\text{s}} = \frac{600\text{m}}{600\text{s}} = 1\ \frac{\text{m}}{\text{s}}$$

Velocity. Velocity describes the speed and direction of an object. Somebody wants to catch Arphy, it is not enough when we say: He left home with the speed 1m/s. We also need to say that he went straight to school. Velocity, like speed, has SI units of metres per second (m/s), but because it is a vector, you must also include a direction. (Furthermore, the symbol **v** for velocity is bold because it is a vector, which contrasts with v for speed.) In our examples, since we are moving in a straight line, we only need to determine the positive direction and then add or subtract speeds accordingly.

Acceleration is any change in velocity. How can velocity change? It can increase or decrease, or there can be a change in direction. Acceleration is the change of magnitude or direction of velocity, or both.
Arphy was playing games late last night, so he did not get up in time. His father took him by car, but last 100 meters, Arphy had to make himself. To come in time, he started running with constant acceleration 0,1m/s^2. Watch the animation.

Suppose for example you travel to your grandparents living 150 km from your home and you will make it in 3 hours. Your average speed will be:

$$v_{average} = \frac{150\text{km}}{3\ \text{hour}} = 50\ \frac{\text{km}}{\text{h}} = 13{,}8\ \frac{\text{m}}{\text{s}}$$

The meaning of average speed is that if it would be possible to travel with the same speed 50 km/h, we will make it in 3 hours. But we know that a car's speed would likely increase and decrease many times over a 3-hour trip. We stopped at the traffic lights; we could have exceeded the speed limit. If the police were to calculate our speed using the same method as we did for average speed, they would not be able to provide concrete evidence. The police must do the calculation of our speed at a specific instant in time, our instantaneous speed. The one we can see on car's speedometer. In fact, we cannot measure it with exact precision, but we can measure the distance traveled in a very short time, so that the speed of the observed object does not change very much (preferably at all) during that period.

Now you can see that his speed increases. In the end his speed was 4,5 m/s . To be able to imagine it better let us change it into km/h.

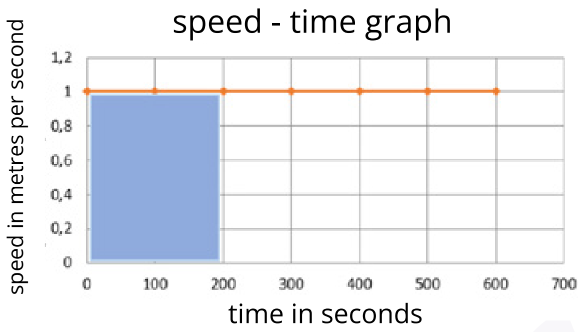
$$4{,}5\ \frac{\text{m}}{\text{s}} = \frac{4{,}5.10^{-3}\ \text{km}}{\frac{1}{3600}\ \text{h}} = 4{,}5.3{,}6\ \frac{\text{km}}{\text{h}} = 16\ \frac{\text{km}}{\text{h}}$$

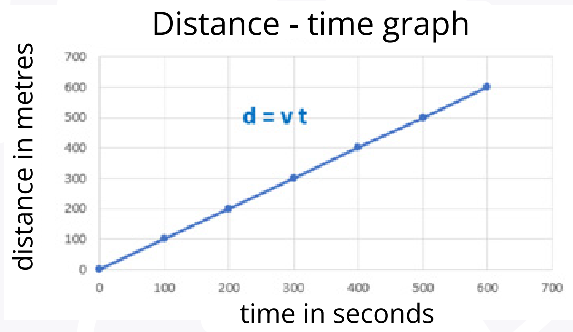
For marathon, which means 42.195 km the best runners have time about 2 hours. That means about 20 km/h. This is average speed. Arphy's average speed was

$$v = \frac{600\text{m}-500\text{m}}{44{,}7\text{s}-0\text{s}} = 2{,}24\ \frac{\text{m}}{\text{s}} = \frac{2{,}24.10^{-3}\ \text{km}}{\frac{1}{3600}\ \text{h}} = 2{,}24.3{,}6\frac{\text{km}}{\text{h}} = 8\ \frac{\text{km}}{\text{h}}$$

16 km/h was the maximum speed Arphy reached. We can learn much more from the last two animations.

Let us work with the graphs from the animations.

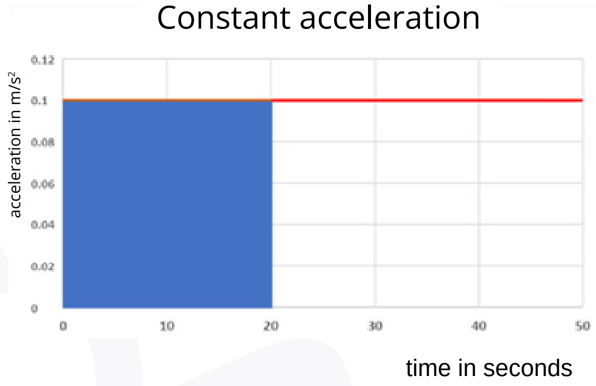


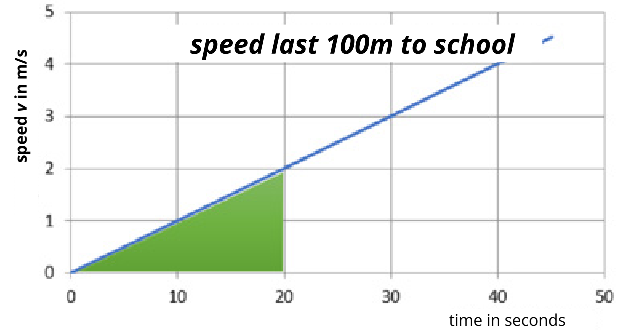


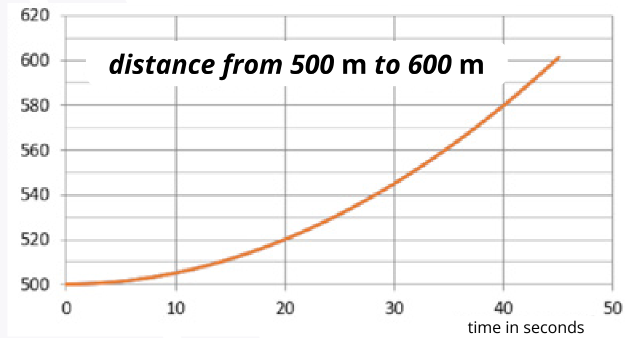
There are two graphs: speed – time, distance - time. Speed is constant and when we want to know the distance made in certain time we can do it by calculating the area under the curve of speed. We will do it for 200 seconds, the area is

$$A \;=\; \text{the area of the blue rectangle} \;=\; 1\,\frac{\text{m}}{\text{s}}\,\text{(speed) 200s (time)} \;=\; 200\text{m}$$

If we look at the distance – time graph at the time 200s the distance on y axis shows 200m.
The equation for calculating the distance done with constant speed is: *d=vt*







There are three graphs: acceleration – time, speed – time, distance - time. Acceleration is constant and when we want to know the speed in certain time we can do it by calculating the area under the curve of acceleration - time graph. We will do it for 20 seconds, the area is

$$A \;=\; \text{the area of the blue rectangle} \;=\; 0{,}1\,\frac{\text{m}}{\text{s}^2}\,\text{(acceleration)20s(time)} \;=\; 2\text{m/s}$$

The equation for calculating the speed done with constant acceleration is: $v=at$
If we want to know the distance we must do the calculation under the curve speed -time graph, as we did it for previous case.

$$A \;=\; \text{the area of the green triangle} \;=\; \frac{2\,\frac{\text{m}}{\text{s}}\,\text{(speed)20s(time)}}{2} \;=\; 20\text{m}$$

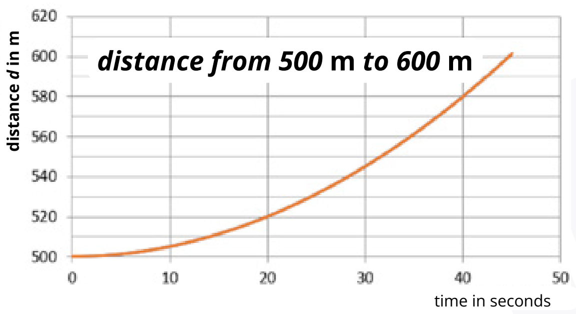
Indeed, from the time Arphy started running at 500 metres he made 20 metres in 20 seconds time. What about the equation?

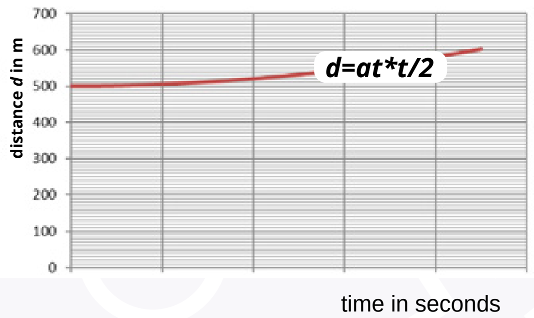
$$d \;=\; \frac{\text{speed time}}{2} = \frac{vt}{2} = \frac{v(v=at \text{ as we derived already})t}{2} = \frac{at\ t}{2}$$

The equation for the distance done with constant acceleration is: $d = \frac{at\ t}{2}$

There is one more thing concerning the graphs. For the plot distance -time when Arphy was running to school we used the starting position 500m and not 0.

See the graphs and you will understand why.



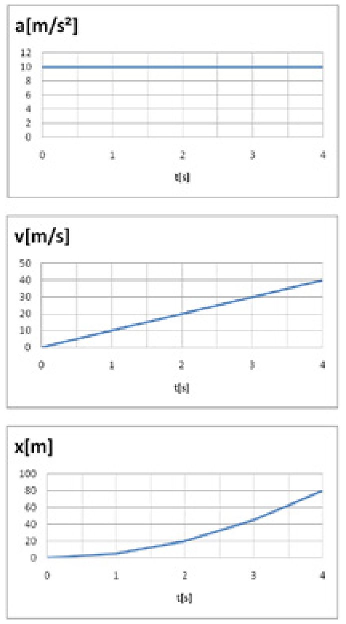


In the first graph we skipped the first 500 metres, where Arphy was in the car. It was not interesting for us. We wanted to know about the last 100 metres, he was running with constant acceleration and the graph was easier to read in such a way. But if somebody asked about the distance from home to school we must add the distance made by car and the distance made by running. As you could see reading graph is important and helpful. We will continue with more examples to understand it better, but before there are some useful hints for solving problems in physics.

Problems solving is an art; there is no simple recipe for obtaining the solutions. Here is an example of step-by step approach to problem solving.

1. Draw a sketch of the situation described in the problem and label all relevant quantities. The information may also be presented in the form of graph, or a table, so try to imagine the story behind it.
2. Then try to visualize the process as though you were watching a movie or imagine how to "tell the story". It is a mistake to immediately search for an answer without thinking about what is happening and what physical principles are governing the process. Story telling is important aspect of studying any science.
3. Prepare a complete list of the given (known) and unknown variables.
4. Choose the equation that matches your variables.
5. Express the unknown quantity from the equation.
6. When you substitute numbers, also include the units of these numbers. The units in your equations should then combine or cancel in such a way as to give the correct units for the result. If the units do not combine or cancel in the expected way, something has gone wrong with your algebra.
7. After you have finished your calculations, always check whether the answer is plausible. For instance, if your calculation yields the result that a diver jumping off a cliff hits the water at 3000 km/h, then somebody has made made an error somewhere!

The following mind map will help you better visualize the concept of rectilinear motion using both formulas and graphs.



a is constant, in our case $10\,\frac{m}{s^2}$

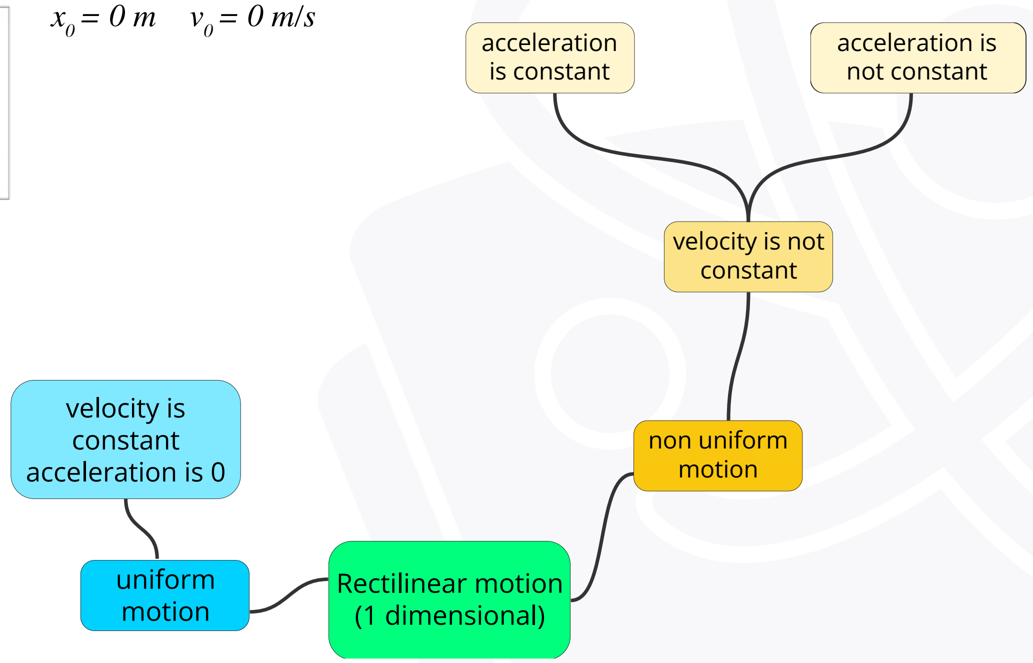
speed = area under acceleration - time curve

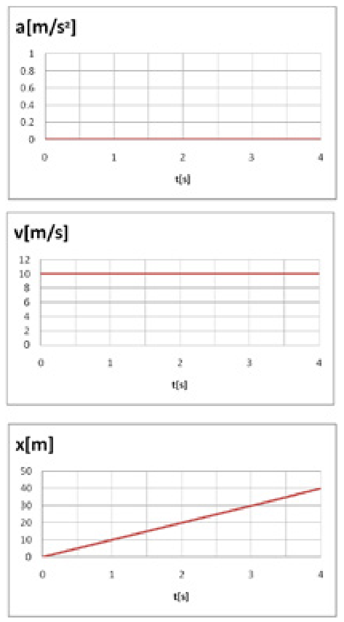
$= \text{in 4 seconds} = 10\,\frac{m}{s^2}\,(a)\ 4s\ (t) = 40\,\frac{m}{s}$

distance = area under speed - time curve =

$\text{in 4 seconds} = \frac{1}{2}\,40\,\frac{m}{s}(v)\ 4s\ (t) = 80\ m$

$x_0 = 0\ m \quad v_0 = 0\ m/s$





AREA=LengthxWidth

Δt=(1s-0s) *v=10m/s*

1s x 10m/s=10m

Δt=(4s-0s)

time in the end *time in the beginning*

Δx= v x Δt

AREA WIDTH LENGTH

x=10m/s x 4s=40m

DISTANCE=AREA UNDER THE SPEED CURVE

Δ - change of variable if initial point is 0 → Δx = x-0=x

Parts of a Graph.

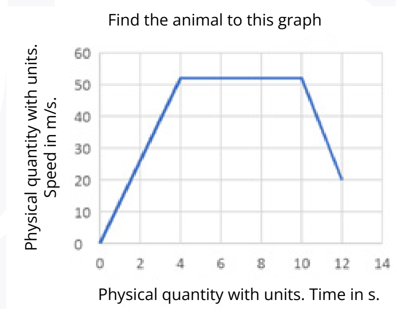
Graphs are utilized in all scientific areas and real-life scenarios as well. We need to know how to construct a graph. We will work with graphs that have 2 axes intersecting at right angles. The horizontal axis is called the x-axis and the vertical axis is called the y-axis. These axes intersect at the origin. We can plot points, lines, and curves on a graph. There should be a title assigned to the graph for easier indication what the graph represents and the most important we must put on each axis the sign, unit, and scale for the physical variable. Any measurement of position, distance, or speed must be done with respect to a reference frame. The significance of all parts of the graph will be explained in the following examples.

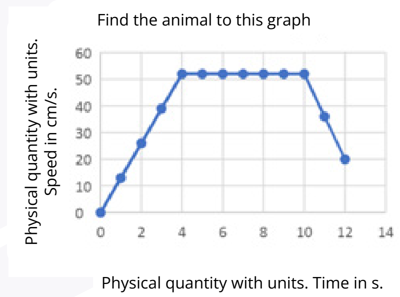
Example;

Speed of animals taken from encyclopedia:

Cheetah: 120 km/h – 33,3 m/s; Gyrfalcon: 187 km/h – 52 m/s; Golden Eagle: 270 km/h – 75 m/s; Garden snail: 0,05 km/h – 0,014 m/s; Three-toad sloth: 0,27 km/h – 0,075 m/s; Slow loris: 1,9 km/h – 0,53 m/s; Gila monster: 2,7 km/h – 0,75 m/s; Pig 18 km/h – 5 m/s.

In the following graphs the names of the animals are missing. Graphs represent the changing speed of the animal with time.





Solve the following tasks, do not change the order:

1.Which graph corresponds to a faster animal? Explain.

2.Find out the titles so that you assign the speed to animals. Use the data from the beginning of the example.

3. Look well at the units.

If you want to compare two objects in speed, size or whatever you must take care of units. Let us repeat how to change speed units

$$\frac{km}{h} = \frac{1000(10^3)m}{3600s} = \frac{1m}{3{,}6s} = 0{,}27\,\frac{m}{s} = 0{,}27\,\frac{100m}{s} = \frac{27cm}{s}$$

Example

The car moves on a straight horizontal road at a constant speed v_0 = 72 km / h. The driver sees an obstacle in the distance of 132 m, a fallen tree, and starts to brake. His reaction time was t_0 = 1.5 s until the brakes became effective. Then the car reduces its speed by 4 m / s every 2 s.

a) On graph paper, draw a graph of the dependence of speed in the car from the moment of seeing the obstacle.

b) Using the graph, determine the time t_1 for which the driver will stop the car from the moment of seeing the obstacle.

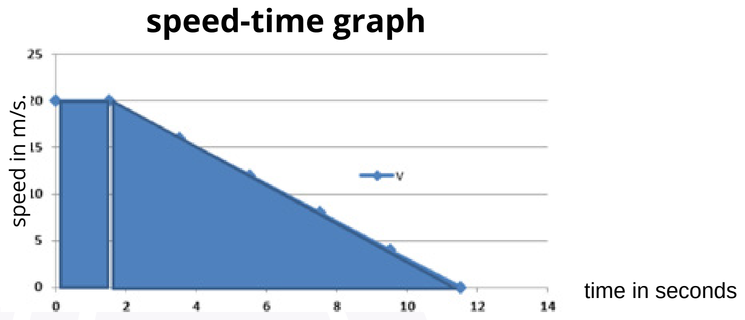
c) Using the graph, determine the path d_1 that the car travels in time t_1. Will it stop in front of an obstacle or hit it?

Further on we will change some factors.

d) If the driver was inattentive and his reaction time increased to $t_{0'}$ = 2,5 s, would he be able to stop in front of the obstacle under the stated braking conditions? Draw a graph for this case, you can use the previous one with different colours, or make a new one.

Solution:

a)



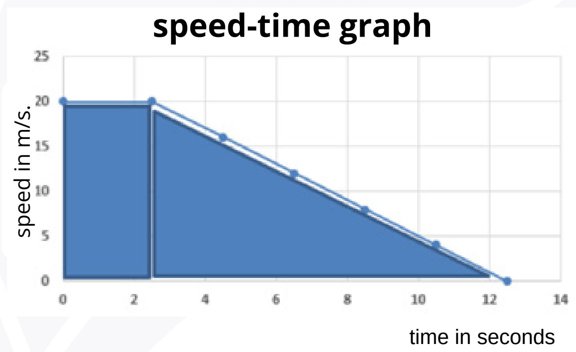
b) Time necessary to stop the car is 11,5 seconds

c) The path (distance) can be calculated as the area under the curve of speed – time graph.

$$\text{Area} = \text{area of rectangle} + \text{area of triangle} = 1{,}5\text{s}\ 20\frac{\text{m}}{\text{s}} + \frac{(11{,}5\text{s} - 1{,}5\text{s})20\frac{\text{m}}{\text{s}}}{2} = 30+100 = 130\text{m}$$

As the distance to the tree was 132 metres, the car stops before hitting it.

d) Solution



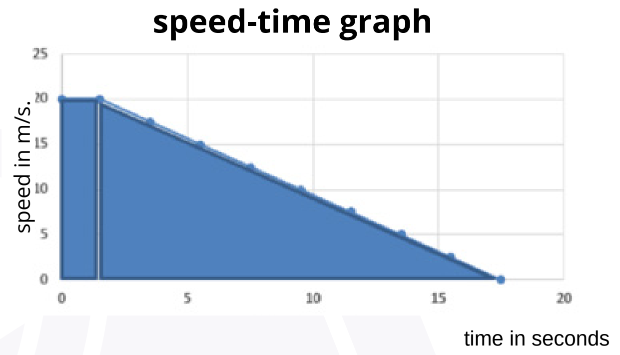
In this case the car stops at 12,5 s. The distance calculated as the area under the curve of the speed-time graph is:

$$\text{Area} = \text{area of rectangle} + \text{area of triangle} = 2{,}5\text{s}\ 20\frac{\text{m}}{\text{s}} + \frac{(12{,}5\text{s} - 2{,}5\text{s})20\frac{\text{m}}{\text{s}}}{2} = 50+100 = 150\text{m}$$

Reaction time one second more (picking up the mobile phone, e.g.) will cause the driver to crash into the tree.

e) Similarly, bad state of brakes or slippery road will change the situation. The reaction time will be 1,5 s. The deceleration of the car during braking in that case would be less, e.g. for every 2 s by 2,5 m / s. Draw a graph for this case in the original coordinates, or make another one.
f) What lessons can be learned by drivers about the technical condition of the vehicle and the road?

e) Solution



In this case the car stops at 17,5 s. The distance calculated as the area under the curve of the speed-time graph is:

$$\text{Area} = \text{area of rectangle} + \text{area of triangle} = 1{,}5\text{s } 20\frac{\text{m}}{\text{s}} + \frac{(17{,}5\text{s} - 1{,}5\text{s})20\frac{\text{m}}{\text{s}}}{2} = 30+160 = 190\text{m}$$

Bad brake condition or slippery road will cause the car to crash.

One more important comment to this example.
In all cases the car was slowing down. We put the information in words:

Then the car reduces its speed by 4 m / s every 2 s.
The deceleration of the car during braking in that case would be less, e.g. for every 2 s by 2,5 m / s.
If car reduces its speed, it means there is a deceleration, which is an expression for acceleration with (-) sign.

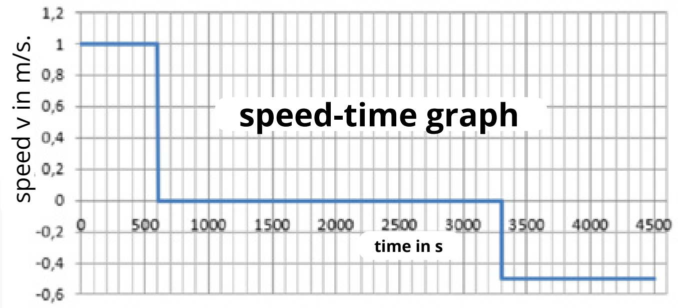
So, reducing speed by 4 m / s every 2 s could have been written as $a = -\frac{4\frac{\text{m}}{\text{s}}}{2\text{s}} = -2\frac{\text{m}}{\text{s}^2}$

For every 2 s by 2,5 m / s could have been written as $a = -\frac{2{,}5\frac{\text{m}}{\text{s}}}{2\text{s}} = -1{,}5\frac{\text{m}}{\text{s}^2}$

f) Your comments, question, observations.

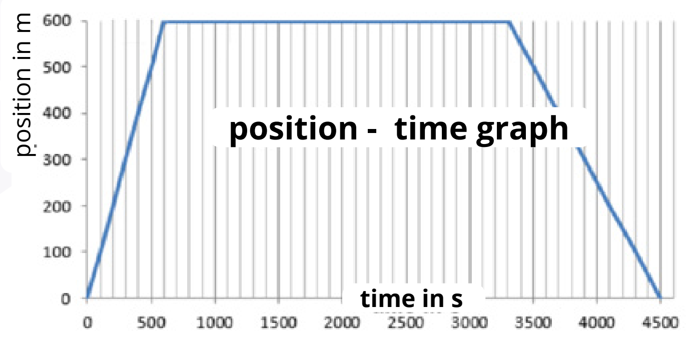
On this page try to put together the situation, when the reaction time increased to 2,5s and the speed will decrease every 2 s by 2,5 m / s.

Example
One day Arphy went to school as usually, his speed was constant 1m/s. But after an hour he felt so bad, that the teacher sent him home to cure himself. As he did not feel well his speed decreased and he walked only 0,5 m/s.
Previously we were talking about the difference between speed and velocity. To be able to make a graph speed – time and distance time you must include the information that Arphy went to school (let us give + sign) and then he went in the opposite direction (- sign).



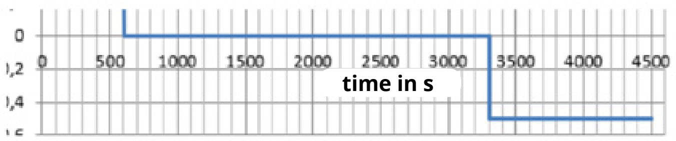
The graph will look like this. Arphy moved on a line, so we must only take care of direction (+ or – sign).

To find out the distance he made, we will put his position on the y -axis and time on the x-axis. The starting point was at home and he returned back home. His position in the beginning and at the end was the same. To calculate the distance we must add the way to school (600m) and the way back home (600m), so altogether he walked 1200 metres. **But displacement, which is the path between the final and starting (initial) point was zero.**



So, as you can see the graphs are more intuitive, then words. If you read the data carefully, you can solve the tasks.
(With the position - time graph, you have to keep in mind what the graph says: it is the position in which Arphy is located on the connection between home and school, namely between 0-600 metres towards the school and then from 600 metres back home.)

And the very last task. Do the calculation of the distance the same way as in the previous example. (The area under the curve of the speed-time graph). What you must take care of is the time axis.



What is the number of the marks between (0s-1000s) and in the same way between (1000s-2000s) (2000s-3000s) and so on? There are 10 of them and it means that each mark means 100 seconds more. If it was all written under the axis, the graph would become cluttered.

If you do not want to work with big numbers, you can measure time in minutes. Then you must change the speed into metres per minute. Graphs will look as follows.

Your comments, question, observations.

Let us read the data:
Arphy started his way at home and this was his initial point. Position was zero and the time was 0 seconds. He was walking with the speed 1m/s. At school the time was 600 seconds, position 600 metres.

He stayed at school one lesson , which is 45 minutes , or 2700 seconds. His position did not change

Arphy started his way home at 3300 seconds. 600 seconds from home to school plus one lesson together 3300 seconds).

Arphy was walking home from 3300 seconds to 4500 seconds, his speed was only 0,5 m/s. Now you can do the math.